Optimization: A Journal of Mathematical Programming and Operations Research

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gopt20

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Version of record first published: 19 Jul 2012.

To cite this article: Nikolaos Goumagias & Dimitris Hristu-Varsakelis (2012): Tax evasion by risk-averse firms in Greece: a discrete Markov-based optimization model, Optimization: A Journal of Mathematical Programming and Operations Research, DOI:10.1080/02331934.2012.707654

To link to this article: http://dx.doi.org/10.1080/02331934.2012.707654

Optimization
2012, 1–15, iFirst

Taylor & Francis
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Tax evasion by risk-averse firms in Greece: a discrete Markov-based optimization model†

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(*Received 31 January 2012; final version received 25 June 2012*)

We present a Markov-based model of the process via which a 'representative' Greek risk-averse firm decides the degree to which it should engage in tax evasion. The model is constructed around a simplified version of the Greek tax system which includes random audits and penalties for under-reporting profits. For its part, the firm is allowed to manipulate its stated profits, potentially exposing itself to future penalty payments, in an attempt to maximize the expected utility of its after-tax wealth. Using our model, we determine the optimal behaviour expected of the firm as a function of the parameters of the tax system, and identify subsets of the audit probability – tax penalty space which 'remove' the inventive for tax evasion. This allows us to – among other things – evaluate the effectiveness of the parameter values currently in use and determine the implied level of risk-aversion for the average Greek firm.

Keywords: optimal taxation; Markov chains; dynamic programming; discrete optimization; Greece

AMS Subject Classifications: 60J10; 37N40; 91B99; 01A20

1. Introduction

In the grip of a dire financial crisis, Greece is currently the focus of ECB and IMF intervention. As a result, a series of austerity measures have been instituted in order to ostensibly reduce and eventually transform the current budget deficits into surpluses. An important part of the effort has to do with the reinforcement of tax revenues and the reduction of tax evasion in Greece, the latter being a well-known and persistent problem. It is widely accepted that Greek firms (we will use the word 'firm' to mean incorporated entities in Greece, or elsewhere, that operate following the general accounting principles commonly known in Europe as S.A. (Société Anonyme)) can – and often do – engage in tax evasion by understating profits (either inflating expenses, or keeping revenue off-book). One of the ways the government

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†This article is an extended version of the work presented at 10th Balkan Conference and 1st International Symposium of Operational Research, 2011.

ISSN 0233–1934 print/ISSN 1029–4945 online

http://dx.doi.org/10.1080/02331934.2012.707654
http://www.tandfonline.com

tries to prevent this from happening is via an audit–penalty mechanism, which (randomly) scrutinizes tax returns, normally for a period of up to 5 years in the past, and imposes penalties for any irregularities discovered. However, the effectiveness of this mechanism is widely questioned.

The purpose of this article is to explore the problem of tax evasion by Greek firms via a Markov-based model that will be suitable for making quantitative as well as qualitative statements about the current state of affairs, and that may be useful in 'tuning' the system. Our model will incorporate the basic attributes of the Greek taxation system (described in more detail in the next section) from the firm's perspective, including the effects of tax penalties and audit probabilities under different levels of risk aversion. Specifically, we consider a 'representative' firm which evolves in a Markov chain-like environment whose state corresponds to the firm's tax status (e.g. the number of years since its last audit). The firm aims to maximize the utility of its future wealth by choosing to conceal a fraction of its profits each year. Here, the word 'future' refers to a time 5 years after a tax evasion decision is made by the firm, that is, how long the government retains the right to an audit (and thus to possible penalties against the firm). Using our model, we produce 'maps' of tax penalty and audit probability pairs for which the firm has an incentive to conceal its profits, and find the degree to which it would do so. We will assume that the firm is risk-averse with constant relative risk aversion coefficient. Our model is useful for calculating the relative efficiency of tax penalty and audit probability as tax evasion deterrents. By knowing how the firm reacts when these parameters are changed, the government can then attempt to optimize them, taking into consideration the firm's level of risk aversion. A further contribution of this work is the estimation of the current level of implied risk aversion for a typical Greek firm, something which could be useful for designing an effective tax policy mix.

This article is structured as follows. Section 1.1 contains a brief literature review. In Section 2 we present the description of our model, including the firm's reward function we aim to maximize. In Section 3 we present the results of our model. We map the optimal decisions of the firm in the audit probability–tax penalty space in graphical form, under different levels of risk aversion and discuss their implications for government's revenues.

1.1. *Related work*

In this section we presenting a brief discussion of previous quantitative studies on tax evasion and optimal taxation. According to [24], the term 'optimal taxation' can take on different meanings. It may involve the minimization of resources used to collect taxes, fairness and justice of the tax system (e.g. direct *vs.* indirect taxation), or economic efficiency. Early approach on the subject begun in the nineteenth century. However, interest in optimal taxation was minimal until the late 1950's [18].

In broad terms, one may identify three main approaches in the literature. One has to do with the construction of simple (usually static) models in which one looks for an equilibrium between the actions of a taxpayer on one side, and the government, or tax authority (TA), on the other. Early work along those lines begins with [1,28], among others. The first of these works introduced a model based on portfolio allocation, where the taxpayer decides the optimal allocation of her/his gross income between a risky asset (the undeclared income) and a risk-free asset (the disclosed income). The authors examined the decision process of an individual taxpayer who may avoid taxation by under-reporting, and discussed the effect of tax rates on the level of tax evasion. That work included both static and dynamic aspects of the decision to evade income taxes. As Andreoni et al. [2] later argued, the model of [1] had several drawbacks, such as the assumption that audit probabilities remain stable, and that its results did not agree with the empirical evidence. A number of later papers introduced the *morality* of taxpayers as a factor (e.g. Frey and Feld [9] and Gordon [11] who also included the actions of auditors as a variable). The morality of the tax payers can also explain why a higher tax rate can potentially increase tax evasion even under decreasing absolute risk aversion. Moreover, Hindriks [15] proposed capturing morality by assigning premiums to auditors that reveal tax evasion, in order to counter the incentive for accepting bribes.

Another portion of the literature seems to have followed some of the suggestions made by Allingham and Sandmo [1] (as well as [28]) and constructed models which included various macroeconomic quantities, such as labour [3]. That work concentrated on the effects of an increase in the probability of detection on the taxpayers' chosen level of tax evasion and found labour to play a significant role. The effect of labour supply was also examined by Cowell [6] and Pencavel [20]. In those works, an increase in the parameters governing tax compliance decreases potential wages and consequently the tax base from which the government collects revenue. A different approach was introduced in [17] that examined the problem of optimal tax policy selection within the rational expectations framework. Apart from labour supply, the authors examined the effect on optimal taxation of consumption and public expenditures as well. They introduced a multi-period model and showed that optimal control techniques were not applicable to the optimal taxation programming problem, because the decisions a taxpayer is called to make today depend on future events (e.g. changes in tax rates).

A third group of papers concerns the so-called 'principal-agent' approach as well as a game theoretic approach to equilibrium. In these works, taxpayers are regarded as risk neutral. The aim is the maximization of the government revenues through tax penalties and tax rates, both of which are modelled as linear functions. The first introduction of the principal-agent framework was by Reinganun and Wilde [21], who presented a simple 'cut-off' model for tax audits. In essence, the government increases the audit probability for taxpayers who report incomes below a cut-off threshold. That idea was pursued further in [23] that explored the conditions under which there is an optimal cut-off strategy. On the other hand, according to Andreoni et al. [2], when the government cannot or is not willing to stick to its audit strategy, the relationship between the tax payers and government can be captured by a sequential game. That approach was taken in [14,22], for example. The major difference between these last two papers is that [14] introduces the threat of an audit even when the taxpayer is honest. A variation of the previous game theoretic model is proposed by Graetz et al. [13], who introduces a group of honest tax payers who do not change their policy. These honest taxpayers seem consistent with the empirical evidence. In general, as mentioned in [2], the principal-agent framework predicts that high-income taxpayers tend to report income near the cut-off, but it is difficult to compare that prediction with empirical evidence. On the other hand, game–theoretic models arguably produce more realistic results, with some taxpayers being honest while others under-report, and tax evasion increases with income. More recent work regarding the principal agent approach is by Slemrod [25], who argued that due to the separation between ownership and control, tax avoidance is not by itself an principal-agent problem. On the other hand, Chen and Chu [4] incorporated a separation of ownership and control, and focused on the efficiency loss of tax rates but not of tax penalties. The relative efficacy of tax penalties, based on the principal-agent framework with the separation of ownership and control, was also examined by Crocker and Slemrod [7]. Finally, Desai et al. [8], examined a three-party game, among government, ownership and control, and argued that higher tax rates may increase return to theft. Their findings were consistent with a series of empirical evidence also included in that work.

Finally, there is a body of literature which follows an empirical approach, focusing mainly on the US. Early work includes [5], that introduced an empirical econometric model for capturing the effect of tax rates on tax evasion, and [11] that found tax rates to have a significant effect as well, but – on the other hand – argued based on empirical evidence – that there are groups of tax payers whose behaviour is not affected by tax rates. As in some of the literature discussed in the beginning of this section, one of the variables is taxpayer morality, along with endogenous reputation costs. Frey and Feld [9] argue that the standard model of tax evasion based on expected utility maximization does not perform particularly well in economic analyses because it excludes morality that tends to underestimate the tax evasion. Finally, in an effort to understand the sometimes contradictory results produced by empirical models, some researchers have performed simulation studies (e.g. [10,26]).

With respect to Greece, in particular, the tax evasion literature (most notably [16,27]) provides some theoretical and empirical discussion but little hard analysis. A recent contribution is the decision support model of [12] studied the optimal tax behaviour of risk-neutral firms, and modelled certain non-traditional aspects of the Greek tax system.

As the literature shows, developing a full-fledged tax optimization model is a non-trivial task. National tax systems are usually built from several components or layers, as various taxes and rules come and go. These layers are often intended to act independently, but may also introduce unintended effects. Moreover, taxation varies significantly between different categories of taxpayers (e.g. individuals, small businesses, corporations). Thus, a workable 'universal' model that incorporates all features of a national tax system could be difficult to construct. Most models to date miss one or more important parameters and are not strongly supported by empirical evidence. There are also arguments that they tend to underestimate tax evasion. Moreover, empirical evidence is limited due to the very nature of tax evasion and the 'hidden' economy.

Our aim in this work is to propose an alternative Markov-based approach, which (a) can be used to simulate the decision process (vis-á-vis tax evasion) of both risk-neutral and risk-averse firms, (b) can estimate the effects of tax penalties and audit probabilities on government and firm revenues as well as on tax evasion, thereby providing a helpful tool when it comes to optimizing tax policy and (c) attempts to bridge the gap between past theoretical models and empirical evidence. Our model takes into account various dynamic aspects of taxation (e.g. time-varying audit probabilities) and is parametric so that it can be tuned by a government agency with access to the appropriate data.

2. Model description

We proceed to describe the core components of our model, in the form of an optimal decision problem that captures the salient features of the Greek tax system. We begin with a brief description of the system, as it pertains to firms.

2.1. *The Greek tax system*

The basic components of the current tax system for incorporated entities are a flat tax rate on profits (currently set at 24%), random audits for identifying tax evaders and monetary penalties for under-reporting income. We consider a firm which, at the end of each fiscal year, must declare its net profit to the government or TA. It is very common, through creative accounting, to manipulate the reported results so as to avoid paying a certain fraction of tax, especially in the presence of systemic inefficiencies. The TA can audit a firm so as to verify the true profit. If discrepancies are discovered, a fine is imposed, based on the amounts concealed plus a penalty amount. That penalty depends on the time elapsed since the offense took place, typically at a monthly rate of 2% for any unpaid tax. Thus, 'old' tax evasion decisions are potentially more costly than the recent ones. Tax penalty amounts are subject to a 2/3 'discount' for prompt settlement once the tax evasion is discovered, and are capped at twice the original tax owed.

There are no published data regarding the audit probability, we estimate it based on informal discussions with officials. As only a very limited number of cases can be audited each year, the government retains the right to audit businesses 'retroactively' for up to 5 years in the past. Any tax evasion activity beyond that horizon is essentially capitalized by the firm. As a result, the audit probability is comparatively higher for firms which have not been audited for at least 4 years (and thus have tax statements which are about to go beyond the statute of limitations). We will discuss the audit probability distribution in Section 2.3.

The inefficiencies of the audit mechanism have led the Greek state to periodically employ a policy called 'closure'. At various time intervals (approximately every 5 years), the government provides the option for firms to pay a fixed additional tax in exchange for the firm's tax declaration to be considered truthful and removed from the audit pool. Recent work [12] has showed that the closure option seems to encourage tax evasion, and there is currently pressure being applied on the Greek government to eliminate this policy. For this reason, we will not consider it further.

2.2. *The model*

We define a Markov process whose state set, $\mathcal{S}$, describes the possible tax status of a representative firm in any given year. In particular,

$$\mathcal{S} = \{A, N_1, \ldots, N_4\}, \tag{1}$$

where

- A: the firm is being audited,
- N_j: the firm's last audit took place $j = 1, \ldots, 4$ years ago,

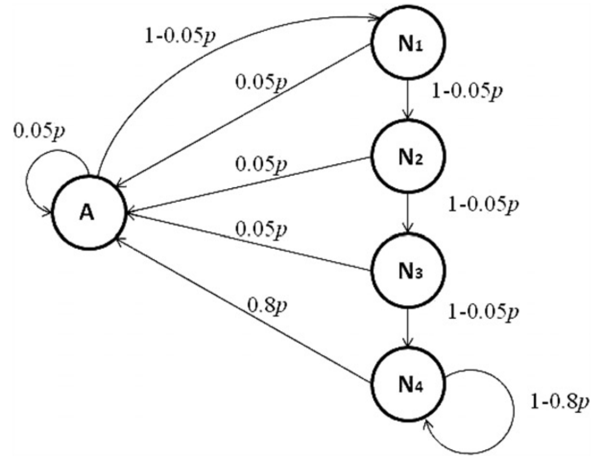

Figure 1. Markov transition diagram for our simplified model of the Greek tax system as it pertains to firms.

When convenient, we will refer to elements of $\mathcal{S}$ by their order of appearance, i.e. the 1st through 5th states, instead of $A, \ldots, N_4$.

The transition probabilities p_{ij} with $i, j \in S$ of moving from state i to j are shown in Figure 1, where p is the overall 5-year audit probability (5%).

While at state $j = 1, \ldots, 5$, the firm will have to decide what fraction, u, of its profits should be concealed. We will consider the firm's tax evasion decisions in the context of a portfolio $W(u,j)$ containing (i) a risk-free asset, R, whose payoff is the declared fraction of profits minus the tax due and (ii) a risky asset, $B(u,j)$, whose payoff is either the discounted undeclared fraction of profits or – in the case of an audit – the tax penalty imposed:

$$W(u,j) = (1-u)R - uB(u,j). \tag{2}$$

The value of the portfolio depends on the state j because the probability of being audited (and thus the value of the risky asset) depends on the number of years since the firm's last audit. If I is the firm's annual profit, r the tax rate and $\beta > 0$ the per-annum tax penalty coefficient, then based on our description of the tax system, we can write:

$$B(u,j) = \begin{cases} Iu & \text{initial state is } j \text{ and audit does not happen in } n \text{ years} \\ I(1 - r - 3/5n\beta r) & \text{initial state is } j \text{ and audit happens in } n \text{ years} \end{cases}. \tag{3}$$

We assume that the firm has constant relative risk aversion, so that its utility function is of the form:

$$U(x) = \frac{x^{1-\lambda}}{1-\lambda}, \tag{4}$$

where λ is the risk aversion coefficient of the 'average' firm and will have to be estimated. We will have more to say about this in the next section.

The firm's objective is then to make the optimal choice regarding the fraction of the profits that it should conceal, so as to maximize the expected utility of the portfolio:

$$G(u,j) = \max_u \left\{ \mathbb{E}(U(W(u,j))) \right\}. \tag{5}$$

We note that, because of the 5-year statute of limitations on tax statements, the firm's current decision will affect its future cash flows for up to 5 years. This is because, in Greece, the audit process goes back up to the firm's previous audit or 5 years, whichever is less. We conclude that the firm's behaviour will depend on the Markov process' first passage probabilities from any state to the first (audit) state, since it is those probabilities that will ultimately determine the expected utility of the firm's decision. More specifically, let $f_{ij}^{(n)}$ stand for the probability of the firm, starting at a Markov state i will reach state j for the first time in n steps. In our case, we assume that the firm begins its economic life in the audit (A) state, without a history of past transgressions. It is well-known that, as in [19], the probabilities $f_{ij}^{(n)}$ satisfy the following system of linear equations:

$$\alpha_{ij}^{(n)} = \sum_{r=1}^{n} f_{ij}^{(n)} \alpha_{ii}^{(n-r)}, \tag{6}$$

where the $\alpha_{ij}^{(n)}$ are the elements of the n-th power of the Markov transition matrix corresponding to the process in Figure 1. The numerical values for $f_{ij}^{(n)}$ are included in the Appendix. Based on the above equation, Equation (5) can be rewritten as follows:

$$G(u,j) = \max_u \left\{ \sum_{n=1}^{5} f_{1j}^{(n)} U(\gamma^n I(1 - r - 3/5n\beta r)) + \left(1 - \sum_{n=1}^{5} f_{1j}^{(n)}\right) \{U(\gamma^5 Iu)\} \right\}, \tag{7}$$

where $0 < \gamma < 1$ is a discount coefficient capturing the time value of money.

Remark We are aware of the fact that it is technically possible for a negative wealth (argument of U) to occur, i.e. when the firm conceals all of its profit and is audited 5 years later with a high penalty factor β. A negative wealth value would be problematic in the context of the utility function chosen. To circumvent this problem, we make the – quite reasonable – assumption that positive wealth will always be preferred over negative wealth, and thus the firm would never consider values of u leading to negative wealth. After applying this restriction, that the maximum in Equation (7) will always occur at some positive wealth value because there is always a u that produces positive wealth (e.g. $u = 0$ – declare all and keep the after-tax profit).

2.3. *Assumptions and parameter selection*

Our model includes a few assumptions which require justification. For simplicity, we will assume that the firm's annual profit, I, is constant throughout its economic life. It is straightforward to allow I to rise at a steady rate, by manipulating the discount coefficient γ. In the following, we assumed an interest rate of 3%, corresponding to $\gamma = 1/(1 + 0.03) = 0.9709$.

Regarding the choices of I and u, we will always refer to 'relative' amounts, so that, for example, $I = 100$, and u is the *percentage* of current profit to be hidden from the authorities. We chose this approach because the firm's decisions are based on its true profit, which the government does *not* know. Expressing Iu as a fraction of firm's hidden profits will make it easier to draw conclusions as to the effectiveness of tax measures and behaviour of the firm. Furthermore, given an estimate of the size of the country's 'hidden economy' (studies such as [16] place it at around 40% for Greece), the quantities computed by the model can be converted to estimates of absolute amounts.

The fraction of the profits that a firm can choose to hide can take values $u \in [0, 1]$. There are several ways that a firm can achieve this, such as overstatement of expenses or income under-reporting. However, it is not practically possible for a firm to hide *all* of its profits from the government due to a number of reasons. First, the shareholders pressure the firm's management to show profits either in the form of dividends or capital gains. Profits also help secure needed loans. Finally, a number of safeguards exist in the accounting system to prevent firms from under-reporting income. We approach this issue here in two ways: we could set some upper bound $u_{\max} < 1$ so that a firm cannot hide more than $u_{\max} I$ of its profits. This, however, assumes knowledge of the firm's true profits. Instead, we let $u_{\max} = 1$ and interpret our results in a 'marginal' way: u will stand for the fraction of profits that a firm would attempt to hide from the government, given the opportunity, because it would be economically advantageous to do so.

Our model can easily be used to also examine the effects of applied tax rates and audit probabilities, however, these quantities will be kept fixed to their estimated current levels. We do this because of space considerations, and because, in the case of audits, for example, an increase is not easy to implement (e.g. it may require hiring of new personnel, training). We will discuss only income tax and ignore VAT collection and payments by the firm, which are subject to a separate mechanism and can be incorporated in the model at a later stage.

With respect to the audit probability distribution, there is a scarcity of official reports. In order to demonstrate our model, we have estimated the various parameters of interest using other sources, including reports in the Greek financial press, which suggest that audits can cover no more than approximately $p = 5\%$ of all firms in a given year. Based on these, we assumed an audit probability of 0.05. This probability is distributed heavily towards firms with past tax declarations whose statute of limitations is about to expire, i.e. a 0.0025 probability that the firm audited is drawn from those with 1–4 years since their last audit, and a 0.04 probability that it is one of those which have not been audited for 5 years.

3. Solving the model

Starting with a risk-neutral firm ($\lambda = 0$), we mapped the level of tax evasion *versus* tax penalty and audit probability (Figure 2). The black area represents 100% tax evasion (i.e. the firm should try to conceal as much of its profit as possible), while the white area corresponds to β, p combinations that lead to full profit disclosure. We observe that tax evasion cannot be eliminated, even with tax penalties of 10 times the tax owed, unless the audit probability is increased above 5%. Moreover, no matter

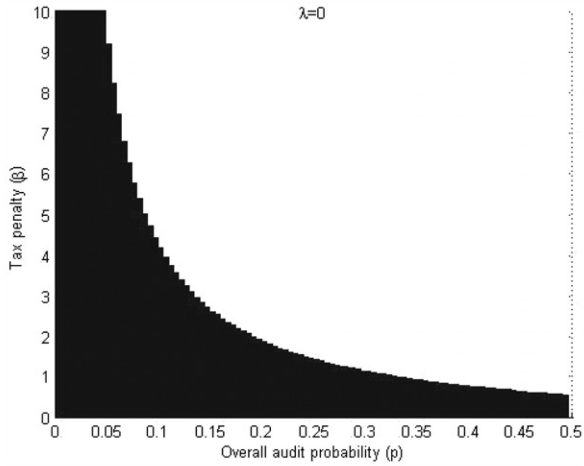

Figure 2. The level of tax evasion as a fraction of a risk-neutral firm's profits.

the audit probability, the tax penalty should be increased to at least 1.5 instead of the current 0.24, in order for tax evasion to begin to decrease. We also note in Figure 2 that under the current combination of audit probability (5%) and tax penalty $\beta = 0.24$, a typical risk neutral firm will choose tax evasion.

3.1. *Risk-aversion*

We have estimated that if the current level of Greek 'hidden' economy persists (40% based on [16]), the risk aversion level of a typical Greek firm should be approximately $\lambda = 6$, because it is for that value of the risk aversion coefficient that the current tax penalty coefficient $\beta = 0.25\%$ and overall audit probability $p = 0.05$ produce tax evasion of approximately 40%. In the following, we discuss the behaviour of $\lambda = 6$ firms, as well as firms which are less risk-averse ($\lambda = 3$).

Figures 3 illustrates the effectiveness of tax penalty increases for firms which are slightly risk-averse ($\lambda = 3$). It is still not possible to completely eliminate tax evasion using 'reasonable' penalty factors when the audit probability level is lower than 5% (Figure 3, however tax evasion is being limited at a tax penalty lower than 30%. For the current levels of $\beta = 0.24$ and $p = 0.05$, a the firm would still choose to hide its profits as much as possible (i.e $u = 1$).

3.2. *Firms with $\lambda = 6$*

For a risk aversion coefficient of 6 (corresponding to the estimates of Greece's 'hidden economy' from [16]), tax evasion begins to deteriorate when the tax penalty reaches 34%, at an audit probability of only 1%. It is obvious from our data that raising the audit probability is a much more effective tax evasion deterrent compared to increasing penalties. The results are shown in Figure 4. Furthermore, under the current levels of tax penalty and audit probability, the level of tax evasion is approximately 41.35%.

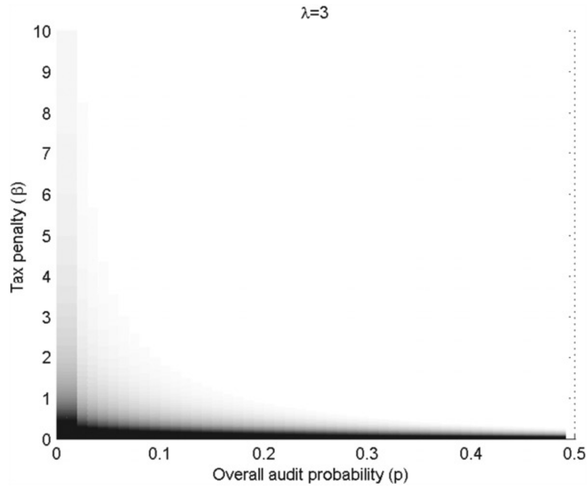

Figure 3. Tax evasion mapping for $\lambda = 3$. The black area represents full tax evasion and white area full profit disclosure. Gray levels represent intermediate choices.

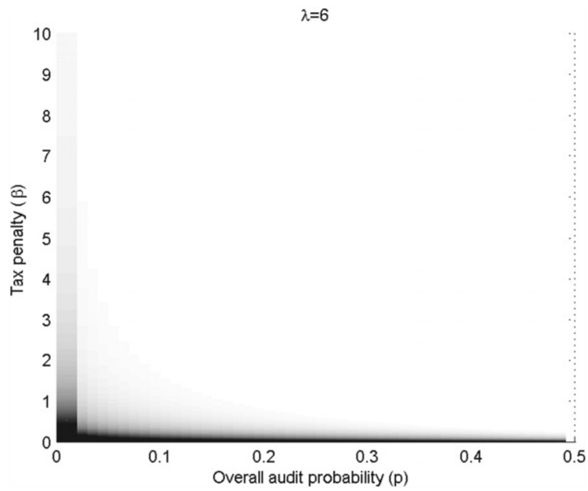

Figure 4. Firm's decision on the tax penalty–audit probability space. Black: 100% tax evasion ($u = 1$); white: full profit disclosure ($u = 0$); Gray levels: intermediate values of u.

3.3. *Government revenues*

We used our model to compute the expected government revenues under different assumptions concerning the firm's risk aversion. If, as the literature suggests, tax evasion in Greece is between 25% and 40%, then, based on our model, the firm's risk aversion coefficient must be between $\lambda = 6$ and $\lambda = 12$. Figures 5 and 6 show the corresponding government revenues per firm as a fraction of the firm's net profits. We observe that the maximum expected revenue of the government cannot exceed

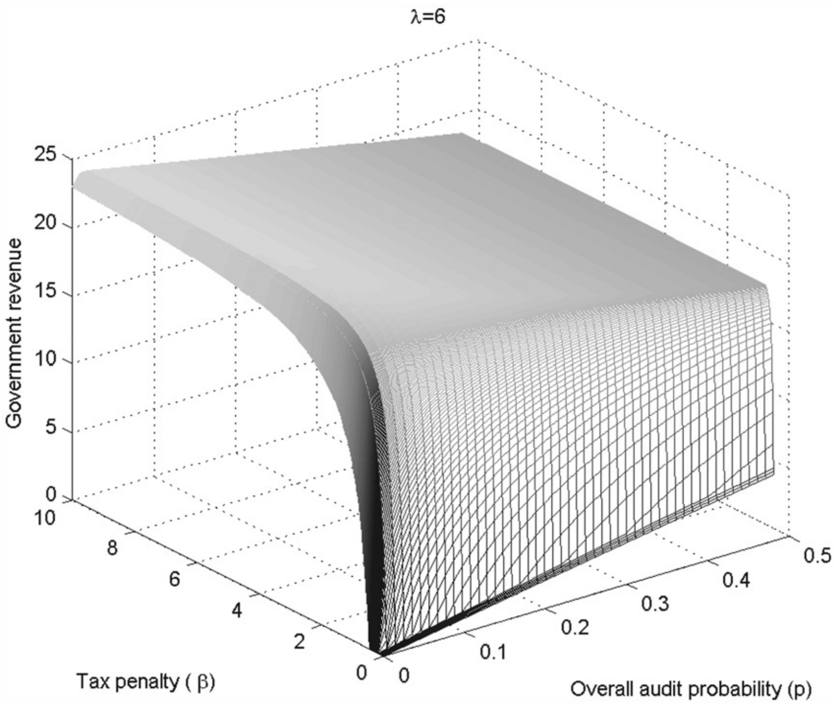

Figure 5. $\lambda = 6$. Government revenues as a function of audit probability and tax penalty coefficient.

the tax rate of 24%. This can be attributed to the fact that the best-case scenario for the firm to disclose the full amount of its profits and thus pay its taxes as it should. Consequently, the combination of tax penalty coefficient and audit probability has to yield an expected expense that does not exceed the normal amount of taxes due, assuming that the firm disclosed its profits fully. Moreover, there is a positive relationship between the tax penalty and government revenues, either by collecting penalties on tax evasion or by preventing tax evasion in the first place. For low audit probabilities the effect of tax penalty is limited.

The differences in tax revenues between the $\lambda = 6$ and $\lambda = 12$ cases are highlighted in Figure 7, with a different view of the same surface shown in Figure 8. The two graphs zoomed in are the are of tax penalties between 0% and 20% and audit probabilities 0% and 10% to make the effect on tax evasion of these two parameters more clear. We note that as risk aversion increases, the government revenues increase as well in the area of tax penalty and audit probability that leads to tax evasion. When the audit probability or the tax penalty are near zero, there is no difference in government revenues for the two cases. It is also interesting to note the sharp, downward 'fold' present in Figures 7 and 8 show that the government misses the most tax revenue (from the less- *vs.* the more risk-averse firms) for a range of tax penalty β between 0.03 and 0.12, which (unfortunately) is close to the current penalty factor.

We have argued that according to the estimated current levels of tax evasion in Greece, the risk aversion coefficient was estimated to be approximately $\lambda = 6$.

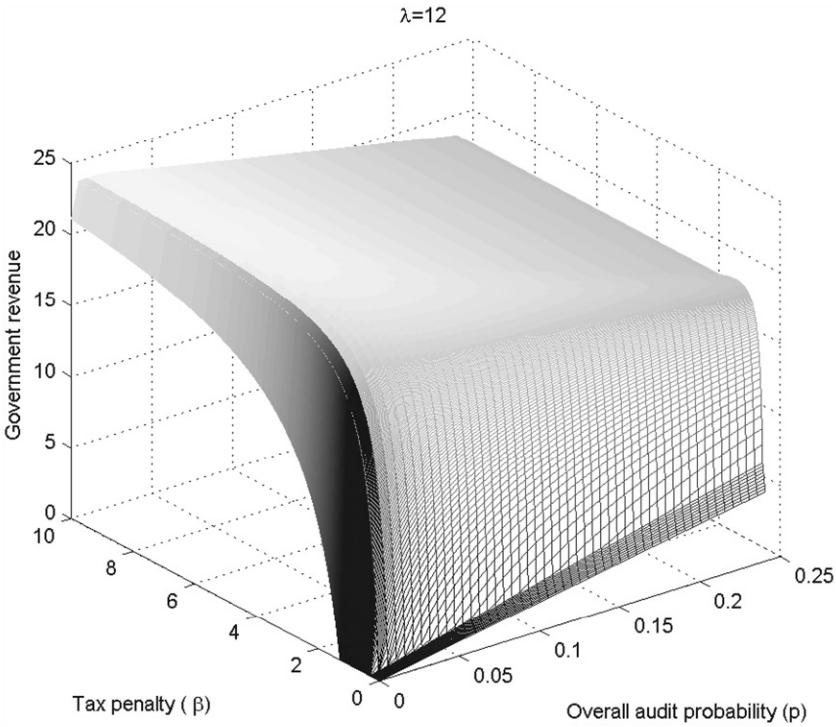

Figure 6. $\lambda = 12$. Government revenues as a function of audit probability and tax penalty coefficient.

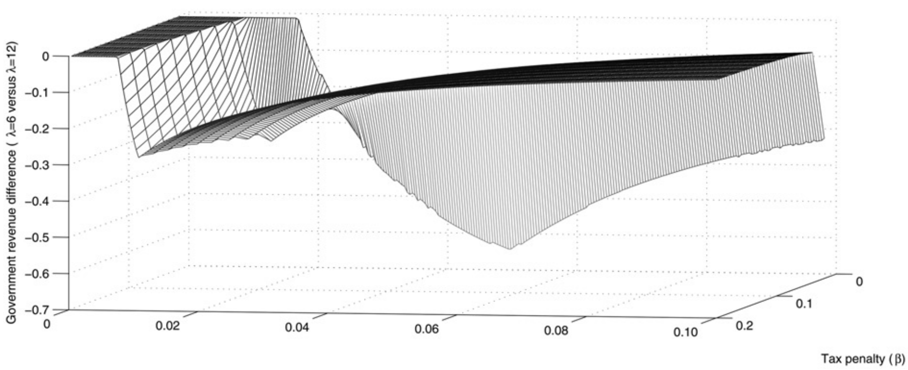

Figure 7. Government revenue collected from a firm with risk aversion $\lambda = 6$ minus that collected for $\lambda = 12$.

Tables 1 and 2 present the level of tax evasion in a *ceteris paribus* form, by keeping the tax penalty stable at its current level of 0.24 and gradually increasing the audit probability, and on the other hand by keeping the overall audit probability at 5% (the current levels of Greek taxation system) and gradually increasing it.

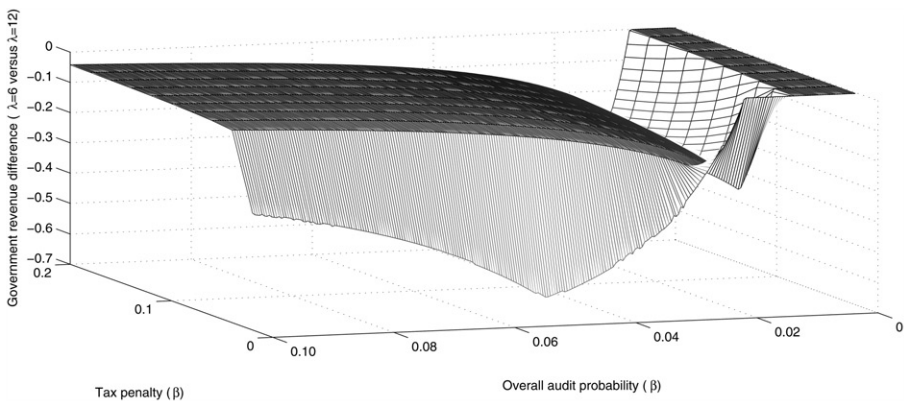

Figure 8. Difference in government revenues collected from a firm with risk aversion $\lambda = 6$ *vs*. those collected from a firm with $\lambda = 12$.

Table 1. Level of tax evasion, u, on behalf of a typical risk-averse firm ($\lambda = 6$), keeping the tax penalty stable at $\beta = 0.24$ for different values of audit probability ($p \in [0.01, 0.20]$).

p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
u	1	0.7571	0.6511	0.5861	0.5384	0.5005	0.4689	0.4417	0.4177	0.3962
p	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
u	0.3767	0.3588	0.3422	0.3267	0.3122	0.2986	0.2856	0.2733	0.2616	0.2503

Table 2. Level of tax evasion, u, on behalf of a typical risk-averse firm ($\lambda = 6$) when keeping the audit probability fixed at $p = 0.05$, for different values of tax penalty ($\beta \in [0, 1.9]$). For $\beta > 1.9$, tax evasion is severely diminished.

β	0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
u	1	1	0.5964	0.3964	0.2914	0.2300	0.1754	0.1466	0.1222	0.1051
β	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90
u	0.0899	0.0776	0.6666	0.0584	0.0514	0.0455	0.0404	0.0359	0.0320	0.0286

This facilitates the comparision of the relative effectiveness of audit probability and tax penalty as tax evasion deterrents.

One conclusion that can be safely drawn from Section 3.3 is that both audit probabilities and tax penalties are relevant tax deterrent tools though their relevance does not remain stable across the board. It seems that the government can restrain tax evasion more effectively through an audit mechanism than employs higher levels of tax penalties.

4. Conclusions

We proposed an optimization tool which can be used to explore the tax behaviour expected of a typical risk-averse Greek firm under various tax penalty and audit probability combinations, for different levels of risk aversion. We constructed a Markov chain-based model in order to simulate the decision-making process of a typical risk-averse Greek firm, seeking to maximize the utility of its final wealth, based on its current decision regarding the fraction of its profits that it will choose to disclose. We computed the Markov chain's first passage probabilities and used them to estimate the effect of penalties and audit probabilities on the firm's decisions. We also examined the firm's behaviour for different levels of risk aversion. The results of our study support the argument that an efficient audit mechanism cannot be substituted by increased tax penalties. The reasons are twofold. First, the lower audit probability directly limits the expected effect on tax penalty levels and a high tax penalty imposed in the case of a verified offense cannot be easily bearable by a typical firm. Future work includes the incorporation into the model of additional parameters of the Greek taxation system, such as VAT and other indirect taxes or commodity taxes.

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Appendix: First passage probabilities

Table A1. First passage probabilities f_{1j} from each Markov state j to the state 1 (audit).

First passage probabilities $f_{1j}^{(n)}$					
			n		
j	0.0025	0.0025	0.0025	0.0025	0.0396
	0.0025	0.0025	0.0025	0.0397	0.0380
	0.0025	0.0025	0.0398	0.0381	0.0365
	0.0025	0.0399	0.0382	0.0352	0.0324
	0.0400	0.0369	0.0340	0.0314	0.0290

Table A2. Probabilities that an audit will not occur within 5 years given that the firm is in Markov state j.

$1 - \sum_{n=1}^{5} f_{1j}^{(n)}$				
0.9504	0.9148	0.8806	0.8518	0.8287