University of Macedonia
Department of Economics
Discussion Paper Series

Input price discrimination with secret linear contracting

Ioannis N. Pinopoulos

Discussion Paper No. 1/2018
Abstract

We study the welfare effects of input price discrimination when an unconstrained upstream supplier uses linear contracts that are unobservable by downstream firms. With homogeneous final goods, we show that banning input price discrimination decreases welfare. This finding is in contrast to that in the existing literature that considers observable linear contracts. When final goods are sufficiently differentiated, it is shown that banning input price discrimination increases welfare. This result is in contrast to that in the existing literature that considers unobservable two-part tariff contracts.

Keywords: Input price discrimination; linear contracts; welfare

JEL Classification: D43; L11; L42
1. Introduction

Price discrimination in intermediate-good markets is an important issue in competition policy. The welfare effects of input price discrimination have long been discussed among economic theorists. An important string of contributions on that issue has focused on linear vertical contracts. A key insight from these contributions is that an unconstrained upstream supplier optimally discriminates among downstream firms based on differences in their derived demands. With linear final-good demand, the more cost-efficient downstream firm has a less elastic derived demand and thus pays a higher input price than its less efficient rival (DeGraba 1990; Yoshida 2000). Under a ban on input price discrimination, the resulting common input price lies between the otherwise prevailing discriminatory prices. Total output remains unchanged (a result due to demand linearity), however, a larger share of this total output is now shifted to the more efficient downstream firm. Therefore, banning input price discrimination increases welfare.

A key feature of the analysis in DeGraba (1990) and Yoshida (2000) is that discriminatory linear contracts are observable by downstream firms. Contracts are usually observable when upstream agents are unions. When upstream agents are firms, however, the assumption of unobservable contracts is much closer to reality. Thus, in this note, we largely adopt the same setup as in DeGraba (1990) and Yoshida (2000), however, we assume that discriminatory linear contracts are unobservable by downstream firms. We show that banning input price discrimination decreases welfare.

When input price discrimination is practiced, the upstream supplier suffers from an opportunism problem: once a downstream firm has paid the input price, the supplier has an incentive to offer a lower input price to the other downstream firm. This opportunism problem, however, does not affect the downstream firms’ derived-demand elasticities and therefore, as

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1Although this finding is considered as counter-intuitive by many economists, there exist real-world examples where less efficient, and thus smaller, downstream firms receive a discount from upstream suppliers. For instance, Luchs et al. (2010) studies a dataset of 345 cases in the US, from 1982 to 2010, involving the Robinson–Patman act and finds that in approximately 20% of the cases plaintiffs were large firms (over 50 million in revenue). See also Villas-Boas (2009), where in the coffee market in Germany, big manufactures such as Jacobs charge large retailers (Metro) higher prices than smaller ones (Markant & Edeka).

2Despite being inefficient, simple linear contracts are observed in practice. For instance, Smith and Thanassoulis (2009) report that, in the UK milk industry, trading between milk producers and retailers is conducted through linear contracts. The assumption of unobservable contracts dates back to the seminal work of Hart and Tirole (1990). To the best of our knowledge, the case of unobservable linear contracting has not yet been examined in a model with input price discrimination.
in the case of observable linear contracts, the more cost-efficient downstream firm pays a higher input price than its less efficient rival.

A ban on input price discrimination eliminates contract unobservability and the supplier’s associated opportunism problem (since it requires all firms receiving the same offer) thus pushing input prices upwards. The resulting common input price may lie above or between the otherwise prevailing discriminatory prices depending on the degree of downstream cost-asymmetry. In either case, (i) total output is reduced causing welfare to decrease and (ii) a larger share of the now smaller total output is shifted to the more efficient downstream firm thereby causing welfare to increase. As it turns out, the positive output-reallocation effect is always outweighed by the negative total-output effect implying that banning discrimination is detrimental to society.

By considering the case of unobservable two-part tariff contracts, O’Brien and Shaffer (1994) and Rey and Tirole (2007), also show that banning input price discrimination reduces welfare. However, the extension of this finding to the case of unobservable linear contracts is not as straightforward as it may seem. Under two-part tariffs, downstream firms, regardless of their efficiency, obtain the same input price irrespective of whether input price discrimination is practiced or not. A ban on discrimination increases input prices and reduces total output and welfare, without causing any reallocation of total output between downstream firms. This result holds for both cases of homogeneous and differentiated final goods. Under linear contracting, due to the presence of the positive output-reallocation effect, the finding that a ban on discrimination reduces welfare when final goods are homogeneous may not extend to the case where final goods are imperfect substitutes. Indeed, we find that a ban on input price discrimination decreases welfare when final goods are close substitutes, however, it increases welfare when final goods are sufficiently differentiated.

2. The model

We consider a vertically related industry consisting of one upstream and two downstream firms denoted, respectively, by $U$ and $D_i$ with $i=1,2$. Each downstream firm purchases an intermediate good (input) from $U$, transforms it into a homogeneous final good in a one-to-one proportion, and sells it to consumers whose demand is assumed to be linear, $Q = a - p$.

Downstream firms face constant marginal cost of producing and selling their goods $c_i$ with $i=1,2$. Without loss of generality, we assume that $D_1$ is more cost-efficient than $D_2$. 
Specifically, we set \( c_2 = c_D > 0 = c_1 \) so that \( c_D \) denotes the production cost advantage of \( D1 \). For simplicity, and without affecting the qualitative nature of our analysis, upstream constant marginal costs are normalized to zero.

Under discriminatory input pricing, the timing of the game is as follows. At the first stage, the upstream supplier, simultaneously and secretly, makes each downstream firm a take-it-or-leave-it offer that specifies a per-unit price for the input, \( w_i \). At the second stage, downstream firms compete in quantities. The upstream supplier faces an opportunism problem due to the fact that offers are secret: once a downstream firm has paid the input price, \( U \) has an incentive to offer a lower input price to the other downstream firm. Multiple equilibria can arise in this setting due to the multiplicity of beliefs that downstream firms can form when they receive out-of-equilibrium offers. Following Hart and Tirole (1990) and Rey and Vergé (2004), we assume passive beliefs: when a downstream firm receives an out-of-equilibrium offer from \( U \), it does not revise its beliefs about the offer received by its rival.

When input price discrimination is banned, downstream firms must receive the same input price. In such case, the game unfolds as described above with the exception that now contract offers are observable. We make the following assumption.

**Assumption 1.** \( c_D < \bar{c}_D = \frac{2a}{7} \).

Assumption 1 requires that the production cost advantage of \( D1 \) over \( D2 \) is not too high and guarantees that both downstream firms will produce a positive quantity of the final good in all cases under consideration. For notational reasons, we use superscripts \( D \) and \( U \) to denote, respectively, the equilibrium values under discriminatory and uniform (non-discriminatory) pricing. All proofs are relegated to the Appendix.

### 3. Input price discrimination

We consider first the case where input price discrimination is practiced. With passive beliefs, each downstream firm anticipates that its rival receives the equilibrium offer and thus puts the equilibrium quantity on the market. As a consequence, each firm chooses its quantity in order to maximize its profits:
Quantities at the second-stage subgame respond only to changes in the own input price according to following downstream best-response functions:

\[
q_1^D(w_1) = \frac{a-w_1-q_2^D}{2}, \quad q_2^D(w_2) = \frac{a-(w_2+c_D)}{2}.
\]  

The upstream supplier chooses input prices so as to maximize its profits:

\[
\max_{w_1,w_2} \pi_U = w_1 q_1^D(w_1) + w_2 q_2^D(w_2).
\]  

Solving together the first order conditions of (3), and using (2), we obtain the equilibrium input prices and final-good outputs as:

\[
w_1^* = \frac{2(3a+c_D)}{15}, \quad w_2^* = \frac{2(3a-4c_D)}{15}, \quad q_1^* = \frac{3a+c_D}{15}, \quad q_2^* = \frac{3a-4c_D}{15}.
\]  

It can be easily checked from the first two expressions in (4) that the more cost-efficient downstream firm pays a higher input price. Under observable contracts, the upstream supplier discriminates among downstream firms based on differences in their derived demands. With linear final-good demand, the more cost-efficient downstream firm has a less elastic derived demand and therefore pays a higher input price than its less efficient rival (DeGraba 1990; Yoshida 2000). Given that contract unobservability does not affect the downstream firms’ derived-demand elasticities, this finding extends to the case of unobservable contracts.

### 4. Banning input price discrimination

We now consider the case where input price discrimination is banned. As mentioned earlier, a ban on input price discrimination makes contracts observable. Each downstream firm chooses its quantity in order to maximize its profits:
\[ \max_{q_1} \pi_{D_1} = (a - q_1 - q_2 - w)q_1, \quad \max_{q_2} \pi_{D_2} = (a - q_2 - q_1 - w - c_D)q_2. \] (5)

From the first order conditions of (5), we obtain quantities at the last-stage subgame for any given level of the input price:

\[ q_1^U(w) = \frac{a + c_D - w}{3}, \quad q_2^U(w) = \frac{a - 2c_D - w}{3}. \] (6)

The upstream supplier chooses the common input price so as to maximize its profits:

\[ \max_w \pi_U = w(q_1^U(w) + q_2^U(w)). \] (7)

From the first order condition of (7), and using (6), we obtain the equilibrium common input price and final-good outputs as:

\[ w^* = \frac{2a - c_D}{4}, \quad q_1^* = \frac{2a + 5c_D}{12}, \quad q_2^* = \frac{2a - 7c_D}{12}. \] (8)

**Proposition 1.** Under secret linear contracts, homogeneous final goods and linear demand:

(i) the common input price lies above (between) the otherwise prevailing discriminatory input prices when the degree of downstream cost-asymmetry is relatively low (high),

(ii) banning input price discrimination decreases total output and welfare.

A ban on input price discrimination eliminates contract unobservability thus pushing input prices upwards. The lower (higher) is the degree of downstream cost-asymmetry, the lower (higher) is the difference between discriminatory input prices, and thus the more likely is that the common input price will lie above (between) them. In either case, a ban on input price discrimination implies that (i) total output is reduced causing welfare to decrease and (ii) a larger share of the now smaller total output is shifted to the more cost-efficient downstream firm leading to an increase in welfare. With linear demand, the negative total-output effect always outweighs the positive output-reallocation effect implying that banning discrimination is detrimental to society.
With observable contracts, DeGraba (1990) and Yoshida (2000) show that the common input price always lies between the otherwise prevailing discriminatory input prices and banning input price discrimination increases welfare. Our analysis reveals that these findings do not extend to the case of unobservable contracts: the common input price may well lie above both discriminatory input prices and banning input price discrimination decreases welfare. In both cases of observable and unobservable contracts, forbidding input price discrimination shifts final-good production to the more cost-efficient downstream firm thus enhancing welfare. However, under observable contracts total output remains unchanged whereas under unobservable contracts total output is reduced after the ban. The output reduction in the latter case is strong enough so that the effects of input price discrimination on welfare crucially depend on contract observability.

Finally, we consider the case of differentiated final goods. Consumers’ inverse demand is given by \( p_i = a - q_i - \theta q_j \), where \( \theta \in [0,1) \) measures the degree of product substitutability.

**Proposition 2.** Under secret linear contracts, differentiated final goods and linear demand, banning input price discrimination decreases welfare when final goods are close substitutes, however, it increases welfare when final goods are sufficiently differentiated.

Banning input price discrimination decreases total output for any degree of differentiation. When final goods are sufficiently differentiated, the supplier’s opportunism problem under discrimination is weak: as a result, when discrimination is banned, the negative total-output effect is weak enough to be outweighed by the positive output-reallocation effect implying an increase in welfare. This result is in contrast to that in the extant literature that considers unobservable two-part tariffs (O’Brien and Shaffer 1994; Rey and Tirole 2007). Under two-part tariffs, downstream firms, regardless of their efficiency, receive the same input price irrespective of whether input price discrimination is practiced or not. Thus, there is no output-reallocation effect and a ban on discrimination increases input prices and reduces total output and welfare for any degree of product differentiation.

5. Conclusions

We have studied the welfare effects of input price discrimination when an unconstrained upstream supplier uses linear contracts that are unobservable by downstream firms. With
homogeneous final goods, we have shown that banning input price discrimination decreases welfare. This finding is in contrast to the extant literature that considers observable linear contracts (DeGraba 1990; Yoshida 2000). When final goods are sufficiently differentiated, we have shown that banning input price discrimination increases welfare. This result is in contrast to the extant literature that considers unobservable two-part tariff contracts (O’Brien and Shaffer 1994; Rey and Tirole 2007).

Appendix

**Proof of Proposition 1.** (i) Using (4) and (8), we first show that the common input price cannot lie below both discriminatory input prices, i.e., \( w^{\ast\ast} - w^{\ast}_{2} = 6a + 17c_{D} / 60 > 0 \). We then calculate \( w^{\ast\ast} - w^{\ast}_{1} = 6a - 23c_{D} / 60 \), which is positive (negative) whenever \( c_{D} < (>) \tilde{c}_{D} = 6a / 23 \), with \( \tilde{c}_{D} < \tilde{c}_{D} \). Thus, the common input price lies above (between) the otherwise prevailing discriminatory input prices when the degree of downstream cost-asymmetry is low (high).

(ii) From (4) and (8), it is straightforward that a ban on input price discrimination reduces total output, i.e.,

\[
q^{\ast\ast}_{1} + q^{\ast\ast}_{2} = \frac{2a - c_{D}}{6} = q^{\ast\ast}_{1} + q^{\ast\ast}_{2}.
\]

Total welfare is defined as the sum of consumer surplus and industry profits, i.e.,

\[
TW^{\ast\ast} = [Q^{\ast\ast}]^{\frac{2}{4}} + (1 - Q^{\ast\ast})q^{\ast\ast}_{1} + (1 - Q^{\ast\ast} - c_{D})q^{\ast\ast}_{2},
\]

where \( Q^{\ast\ast} = q^{\ast\ast}_{1} + q^{\ast\ast}_{2} \) and \( k = D, U \). Define \( \Delta TW^{\ast\ast} = TW^{\ast\ast} - TW^{\ast\ast} \). Using (4) and (8), we obtain:

\[
\Delta TW^{\ast\ast} = \frac{76a(a - c_{D}) - 581(c_{D})^{2}}{1800}.
\]

From the above expression, we have that

\[
\Delta TW^{\ast\ast} \bigg|_{c_{D} = 0} = \frac{19a^{2}}{450} > 0, \quad \lim_{c_{D} \to \tilde{c}_{D}} \Delta TW^{\ast\ast} = \frac{2a^{2}}{525} > 0 \quad \text{and} \quad \frac{\partial \Delta TW^{\ast\ast}}{\partial c_{D}} = -\frac{38a + 581c_{D}}{900} < 0,
\]

which establish that a ban on input price discrimination decreases total welfare. □
**Proof of Proposition 2.** When final goods are differentiated, the equilibrium analysis is the same as the one carried out in the main text, so we simply present here the final equilibrium outcomes, which are given by

\[
\tilde{w}_{1}^{\star} = \frac{2[a(4-\theta) + \theta c_{D}]}{16 - \theta^{2}}, \quad \tilde{w}_{2}^{\star} = \frac{2[a(4-\theta) - 4c_{D}]}{16 - \theta^{2}},
\]

\[
\tilde{q}_{1}^{\star} = \frac{a(4-\theta) + \theta c_{D}}{16 - \theta^{2}}, \quad \tilde{q}_{2}^{\star} = \frac{a(4-\theta) - 4c_{D}}{16 - \theta^{2}}.
\]

(A1)

under discriminatory input pricing, and by

\[
\tilde{w}_{1}^{\star} = \frac{2a - c_{D}}{4}, \quad \tilde{q}_{1}^{\star} = \frac{2a(2-\theta) + c_{D}(2+3\theta)}{4(4 - \theta^{2})}, \quad \tilde{q}_{2}^{\star} = \frac{2a(2-\theta) - c_{D}(6+\theta)}{4(4 - \theta^{2})}.
\]

(A2)

under non-discriminatory input pricing. For \( \theta = 1 \), the expressions in (A1) and (A2) become equivalent to those in (4) and (8) respectively. From (A1) and (A2), it can be checked that a ban on input price discrimination decreases total output for any degree of differentiation.

Total welfare is defined as the sum of consumer surplus and industry profits, i.e.,

\[
\tilde{W}^{\star} = \left[ (\tilde{q}_{1}^{\star})^{2} + (\tilde{q}_{2}^{\star})^{2} + 2t\tilde{q}_{1}^{\star}\tilde{q}_{2}^{\star} \right]/2 + (1 - \tilde{q}_{1}^{\star} - \theta\tilde{q}_{2}^{\star})\tilde{q}_{1}^{\star} + (1 - \tilde{q}_{2}^{\star} - \theta\tilde{q}_{1}^{\star} - c_{D})\tilde{q}_{2}^{\star}, \quad \text{with} \quad k = D, U.
\]

Define \( \Delta \tilde{W}^{\star} = \tilde{W}^{D\star} - \tilde{W}^{U\star} \). The sign of \( \Delta \tilde{W}^{\star} \) can be positive or negative depending on the value of \( \theta \). It is easy to check that

\[
\Delta \tilde{W}^{\star} \bigg|_{\theta=1} = \lim_{\theta \to 1} \Delta \tilde{W}^{\star} = \frac{76a(a - c_{D}) - 581(c_{D})^{2}}{1800} > 0,
\]

where the positive sign of the above expression is derived in the proof of Proposition 1. Thus, banning input price discrimination decreases welfare when final goods are close substitutes. Moreover, we have that

\[
\Delta \tilde{W}^{\star} \bigg|_{\theta=0} = \lim_{\theta \to 0} \Delta \tilde{W}^{\star} = -\frac{5(c_{D})^{2}}{64} < 0,
\]
which implies that a ban on input price discrimination increases welfare when final goods are sufficiently differentiated.

References