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Ioannis Pinopoulos

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Department of Economics, University of Macedonia, 156 Egnatia str, 540 06 Thessaloniki, Greece, Fax: + 30 (0) 2310 891292
Vertical integration and upstream horizontal mergers

Ioannis N. Pinopoulos*

Department of Economics, University of Macedonia, 156 Egnatia Street, Thessaloniki, Greece

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Abstract

In this paper, we study upstream horizontal mergers in vertically related markets. A key aspect of our analysis is that one of the merging parties is a vertically integrated firm. We consider a two-tier market consisting of two competing vertical chains, with one upstream and one downstream firm in each chain. We assume that one vertical chain is vertically integrated whereas the other chain is vertically separated. We also assume that the vertically integrated chain is more cost-efficient in its downstream operations than the independent downstream firm. We show that a horizontal merger between the vertically integrated firm and the independent upstream supplier will increase the equilibrium input price and reduce both consumer and total welfare. When an upstream competitive fringe exists, however, the merger still decreases consumer surplus but it may increase total welfare. The latter finding is important since it implies that whether antitrust authorities favor a consumer or total welfare objective can lead to very different conclusions regarding the merger’s desirability.

Keywords: Vertical relations; vertical integration; horizontal mergers; welfare

JEL Classification Codes: L11; L13; L41; L42

* Corresponding address: Ioannis N. Pinopoulos, Department of Economics, University of Macedonia, 156 Egnatia Street, Thessaloniki, Greece. Phone: (+30) 2310-891-663. E-mail address: me0710@uom.gr
1. Introduction

A classic topic of antitrust economics is the welfare effects of horizontal mergers – that is mergers between competitors. Since vertical relations are ubiquitous in real-world markets, it is nowadays widely acknowledged, by both economic theorists and antitrust agencies, that the vast majority of horizontal mergers take place in either the upstream or the downstream sector of vertically related industries.

In this paper, we study upstream horizontal mergers. A key aspect of our analysis is that one of the merging parties is a vertically integrated firm; in other words, one insider party to the upstream merger is also present in the downstream market through a subsidiary. This assumption is primarily motivated by one of the largest oil mergers ever, namely the BP/ARCO merger.¹ In 1999, British Petroleum Amoco (BP) announced its intention to acquire the Atlantic Richfield Company (ARCO). Whereas both BP and ARCO were present in the Alaskan North Slope (ANS) – the upstream market for crude oil -, only ARCO was present downstream in West Coast refining and marketing. Moreover, BP was a major supplier of crude oil to ARCO’s competitors, such as Chevron and Tosco.

As Bulow & Shapiro (2002) comment, the basic downstream antitrust concern in the BP/ARCO merger “was whether the acquisition of ARCO would allow BP to elevate the price of ANS crude oil to West Coast refineries. Ultimately, higher ANS crude oil prices might lead to higher prices of refined products, especially gasoline, on the West Coast.” The main purpose of this paper is to provide a formal explanation of the aforementioned antitrust issue. In addition, we try to address the following important question from a policy perspective: even if the merger decreases consumer surplus, are there any conditions under which it might increase total welfare?

In our baseline model, we consider a two-tier market initially consisting of two competing vertical chains. In each chain, there is a single upstream firm that produces an input which a single downstream firm uses in one-to-one proportion in the production of a differentiated final good. We assume that one vertical chain is vertically integrated whereas the other chain is vertically separated. In the separated chain, the upstream firm makes a take-it-or-leave-it, two-part tariff contract offer to the downstream firm. We also assume that the vertically integrated chain is more cost-efficient in its downstream operations than the independent downstream firm.

¹See Bulow & Shapiro (2002) for a very thorough discussion of the BP/ARCO merger.
We then consider the case where the independent upstream firm and the vertically integrated firm decide whether they will merge or not. This particular merger, when occurs, is clearly horizontal since both firms are present in the upstream market, however, it also has a vertical aspect that lies on the fact that the previously independent upstream firm is the input supplier of the vertically integrated firm’s rival in the downstream market.

With downstream Cournot competition, and under a general demand function, we show that a horizontal merger between the vertically integrated firm and the independent upstream supplier allows the latter to induce a more accommodating behavior downstream by elevating the input price paid by the independent downstream firm and thus shifting sales of the final-good to its downstream affiliate. Ultimately, a higher input price leads to higher final-good prices thus making consumers worse off. Under our modelling structure, in the pre-merger case, all of the vertically integrated firm’s upstream production is directed to its downstream affiliate (captive sales) implying that the merger does not affect concentration in the upstream market: the upstream market is monopolized both in the pre- and post-merger case(s). Therefore, the merger’s negative impact on consumer surplus stems solely from a vertical partial foreclosure effect. This finding highlights the important role of vertically integrated firms in horizontal merger analysis.

We find that the merger is always beneficial for the merging parties and the industry as a whole. Therefore, the effect of the merger on total welfare is a priori ambiguous since it decreases consumer surplus but increases industry profits. On the one hand, the merger decreases total welfare since it decreases the total quantity supplied in the downstream market. On the other hand, the merger has also a welfare-enhancing effect: since it increases the equilibrium input price, sales will be shifted towards the downstream division of the newly merged firm which is more cost-efficient thereby increasing allocative efficiency. By employing a linear demand function, we show that the resulting increase in allocative efficiency due to the merger is not enough to compensate for the fact that total output is reduced and thus total welfare is lower compared to the pre-merger case.

We also modify our baseline model by introducing the presence of an (equally efficient) upstream competitive fringe while still restricting attention, for tractability reasons, to a linear demand function. Our analysis reveals that in the pre-merger case, the independent upstream firm is essentially unconstrained by the presence of an alternative source of supply as it can

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2Vertical foreclosure is partial in the sense that the independent downstream firm pays a higher input price and produces less of the final good in the post-merger case, however, it is not driven out of the market (in which case foreclosure is complete or full).
offer better terms to the independent downstream firm. However, in the post-merger case, the independent downstream firm’s threat of switching to the fringe limits the market power of the merged firm; the merger will still increase the equilibrium input price and decrease consumer surplus but not to the same extent as in the absence of the fringe. More importantly, the presence of the competitive fringe introduces the possibility that the merger will increase total welfare even though consumer surplus decrease. More specifically, it is shown that the merger is welfare-enhancing when final goods are close enough substitutes and/or the cost advantage of the vertically integrated chain vis-à-vis the independent downstream firm is sufficiently high. This finding is important since it implies that whether antitrust authorities favor a consumer or total welfare objective can lead to very different conclusions regarding the merger’s desirability.

There is a large literature on the effects of upstream mergers in vertically related markets. This literature, which begins with the seminal work of Horn & Wolinsky (1988), also includes, among others, Ziss (1995), Inderst & Wey (2003), O’Brien & Shaffer (2005), Milliou & Petrakis (2007) and Milliou & Pavlou (2013). A key feature of the aforementioned studies is that upstream mergers take place in vertically separated industries. To the best of our knowledge, the case where one of the merging parties is a vertically integrated firm has not been formally examined by the existing literature.

The rest of the paper is organized as follows. In Section 2, we introduce the main features of our model. In Section 3, we perform the equilibrium analysis and derive our main results. In Section 4, we extend the baseline model by introducing the presence of an upstream competitive fringe. In Section 5, we conclude the paper.

2. The baseline model

We consider a vertically related market initially consisting of two competing vertical chains. In each chain, there is a single upstream firm, denoted by $U_i$, $i=1,2$, that produces an input which a single downstream firm, denoted by $D_i$, $i=1,2$, uses in one-to-one proportion in the production of a differentiated final good. We assume that one vertical chain is vertically integrated whereas the other chain is vertically separated, i.e., there is the vertically integrated firm $U1-D1$, one independent upstream supplier $U2$ and one independent downstream firm $D2$ (see Fig. 1 below). Marginal production costs in the upstream market are, for simplicity and without loss of generality, normalized to zero. Marginal transformation
costs in the downstream market are denoted by \( c_i, i = 1, 2 \). We assume that \( c_1 < c_2 \), so the vertically integrated chain is more efficient in its downstream operations.

Figure 1. The pre-merger case.

We then consider the case where the independent upstream supplier \( U_2 \) and the vertically integrated firm \( U_1-D_1 \) decide whether they will merge or not. If the merger occurs, the newly merged firm is denoted by \( I \) (see Fig. 2 below). This particular merger is clearly horizontal since both firms are present in the upstream market, however, it also has a vertical aspect that lies on the fact that \( U_2 \) is the input supplier of \( U_1-D_1 \)'s rival in the downstream market.
Figure 2. The post-merger case. $U2$ and $U1-D1$ merge to form firm $I$.

The timing of the three-stage game is as follows. In the first stage, firms $U1-D1$ and $U2$ decide whether to merge or not. In the second stage, the independent supplier $U2$ (if the merger does not occur) or firm $I$ (if the merger occurs) makes $D2$ a take-it-or-leave-it, two-part tariff contract offer; the contract consists of an input price $w$ and a fixed fee $F$. In the last stage, downstream competition takes place a la Cournot.

Suppose that $U(q_1,q_2)$ is a differentially strictly concave utility function and let $q = (q_1,q_2)$. The representative consumer by maximizing $U(q) - pq$ gives rise to an inverse demand system $p_i = p(q_i,q_j)$, $i,j = 1,2$, $i \neq j$, which is twice continuously differentiable. Inverse demands will be downward sloping, $\partial p_i/\partial q_i < 0$, and symmetric cross effects will be negative, $\partial p_i/\partial q_j = \partial p_j/\partial q_i < 0$, implying that final-goods are substitutes. We also assume that the own effect is larger than the cross effect, that is $|\partial p_i/\partial q_i| > |\partial p_j/\partial q_j|$

Our notational convention throughout the paper will be as follows. The superscript $S$ on a variable will be used to denote the pre-merger case, whereas the superscript $M$ will be used to denote the post-merger case.

3. Equilibrium analysis and results

3.1. The pre-merger case

Working backwards, we start by solving the last stage of the game. Firms $U1-D1$ and $D2$ choose simultaneously and independently their final-good outputs to maximize profits:

$$\max_{q_1} \pi_{U1-D1} = p_1(q_1,q_2)q_1 - c_1q_1, \quad \max_{q_2} \pi_{D2} = p_2(q_1,q_2)q_2 - (w + c_2)q_2 - F.$$ 

The first order conditions of the above maximization problems are given by,

$$p_1 + q_1 \frac{\partial p_1}{\partial q_1} = c_1,$$  \hspace{1cm} (1)

and

$$p_2 + q_2 \frac{\partial p_2}{\partial q_2} = c_2 + w.$$  \hspace{1cm} (2)
respectively. We make the following three assumptions:

Assumption 1. \( \frac{\partial^2 \pi_{U1-D1}}{\partial q_1^2} < 0 \) and \( \frac{\partial^2 \pi_{D2}}{\partial q_2^2} < 0 \).

Assumption 2. \( \frac{\partial^2 \pi_{U1-D1}}{\partial q_1 \partial q_2} < 0 \) and \( \frac{\partial^2 \pi_{D2}}{\partial q_2 \partial q_1} < 0 \).

Assumption 3. \( \frac{\partial^2 \pi_{U1-D1}}{\partial q_1^2} + \frac{\partial^2 \pi_{U1-D1}}{\partial q_1 \partial q_2} < 0 \) and \( \frac{\partial^2 \pi_{D2}}{\partial q_2^2} + \frac{\partial^2 \pi_{D2}}{\partial q_2 \partial q_1} < 0 \).

Assumption 1 guarantees that the second order conditions of the above maximization problems are satisfied. Assumption 2 implies strategic substitutability: firms’ best-response functions in the downstream market are downward sloping, i.e., \( dq_i/dq_j < 0 \). Assumption 3 implies that the best-response functions are well-behaved and have slope less than one, \( |dq_i/dq_j| < 1 \), and therefore there exist unique and stable Cournot equilibria.

Solving together (1) and (2), we obtain the last-stage subgame equilibrium final-good outputs and prices as functions of the input price: \( \hat{q}_i(w), \hat{q}_z(w), \hat{p}_i(w) = p_i[\hat{q}_i(w), \hat{q}_z(w)] \) and \( \hat{p}_z(w) = p_z[\hat{q}_i(w), \hat{q}_z(w)] \). As we show in Appendix, these last-stage subgame equilibrium outcomes have the following properties:

\[
\frac{d\hat{q}_i(w)}{dw} > 0, \quad \frac{d\hat{q}_z(w)}{dw} < 0, \quad \frac{d\hat{p}_i(w)}{dw} > 0, \quad \frac{d\hat{p}_z(w)}{dw} > 0. \tag{3}
\]

Next, we determine the equilibrium contract terms – we solve the second stage of the game. The independent upstream firm \( U2 \) uses the franchise fee to fully extract \( D2 \)’s profits,

\[ F = [\hat{p}_z(w) - w - c_z]\hat{q}_z(w), \tag{4} \]

and thus sets the input price so as to maximize,

\[ \max_w (\hat{p}_z(w) - c_z)\hat{q}_z(w). \tag{5} \]

Hence, the input price is chosen so as to maximize the unintegrated vertical chain’s profits. The first order condition of the above maximization problem, after using (2), is given by:
\[
\frac{\partial c[(\hat{p}_2 - c_2)\hat{q}_2]}{\partial w} = (\hat{p}_2 - c_2) \frac{d\hat{q}_2}{dw} + \hat{q}_2 \frac{d\hat{p}_2}{dw} \\
= (\hat{p}_2 - c_2) \frac{d\hat{q}_2}{dw} + \hat{q}_2 \left[ \frac{\partial p_2}{\partial q_1} \frac{d\hat{q}_2}{dw} + \frac{\partial p_2}{\partial q_2} \frac{d\hat{q}_2}{dw} \right] \\
= \left[ (\hat{p}_2 - c_2) + \hat{q}_2 \frac{\partial p_2}{\partial q_1} \right] \frac{d\hat{q}_2}{dw} + \hat{q}_2 \frac{\partial p_2}{\partial q_1} \frac{d\hat{q}_2}{dw} \\
= w \frac{d\hat{q}_2}{dw} + \hat{q}_2 \frac{\partial p_2}{\partial q_1} \frac{d\hat{q}_2}{dw} = 0
\] 

We know from (3) that \( d\hat{q}_2/\partial w < 0 \) and \( d\hat{q}_1/\partial w > 0 \). Therefore, given that \( \partial p_2/\partial q_1 < 0 \), it is straightforward that the equilibrium input price must be negative in order for the last equality in (6) to be satisfied.

**Lemma 1.** In the pre-merger case, the equilibrium input price is always lower than the upstream marginal cost, \( w^* < 0 \).

According to Lemma 1, the input price reflects a subsidy from \( U_2 \) to its respective downstream firm \( D_2 \). The separated vertical chain, via a lower input price, can commit to a more aggressive behavior in the final-good market. The best-response curve of its downstream firm shifts out, and as the best-response curves are downward sloping, this results in lower final-good quantity for the rival integrated chain, and higher quantity and gross profits for the own downstream firm. The portion of these gross profits that is transferred upstream, via the fixed fee, more than compensates the upstream firm for the losses due to subsidization. In order to verify the latter note that, after using (4), we have that:

\[ \pi_{U_2} = w\hat{q}_2(w) + F = [(\hat{p}_2(w) - c_2)\hat{q}_2(w)] > 0. \]

### 3.2. The post-merger case

When the merger occurs, firms \( I \) and \( D_2 \) choose simultaneously and independently their final-good outputs to maximize profits:

\[
\max_{q_1} \pi_I = [p_1(q_1, q_2) - c_1]q_1 + wq_2 + F, \quad \max_{q_2} \pi_{D_2} = p_2(q_1, q_2)q_2 - (w + c_2)q_2 - F.
\]
It is straightforward that the profit maximization problem of \( D_2 \) is unaffected by the merger. However, the newly integrated firm \( I \) has now profits from two sources: the term \([p_i(q_i,q_2) - c_i]q_i\) captures, as in the pre-merger case, profits from sales of the final good, whereas the term \( wq_2 + F \) reflects profits from selling the input to the independent downstream rival \( D_2 \). Since downstream competition is over quantities, firm \( I \) cannot affect its sales of the input upstream by increasing sales of its downstream rival and thus the profit maximization problem in the downstream market is the same as if firm \( I \) maximizes only downstream profits. Therefore, the last-stage subgame equilibrium final-good outputs and prices are the same as in the pre-merger case.

Next, we determine the equilibrium contract terms. The newly merged firm \( I \) uses the franchise fee to fully extract \( D_2 \)’s profits and thus set the input price so as to maximize,

\[
\max_w (\hat{p}_1(w) - c_1)\hat{q}_1(w) + (\hat{p}_2(w) - c_2)\hat{q}_2(w)
\]

(7)

Hence, the input price is actually chosen so as to maximize overall industry profits. The first order condition, after using (1) and (2), is given by:

\[
\frac{\partial \pi}{\partial w} = (\hat{p}_1 - c_1)\frac{d\hat{q}_1}{dw} + \hat{q}_1 \frac{d\hat{p}_1}{dw} + (\hat{p}_2 - c_2)\frac{d\hat{q}_2}{dw} + \hat{q}_2 \frac{d\hat{p}_2}{dw} = \\
= (\hat{p}_1 - c_1)\frac{d\hat{q}_1}{dw} + \hat{q}_1 \left[ \frac{\partial p_1}{\partial q_1} \frac{d\hat{q}_1}{dw} + \frac{\partial p_1}{\partial q_2} \frac{d\hat{q}_2}{dw} \right] + (\hat{p}_2 - c_2)\frac{d\hat{q}_2}{dw} + \hat{q}_2 \left[ \frac{\partial p_2}{\partial q_1} \frac{d\hat{q}_1}{dw} + \frac{\partial p_2}{\partial q_2} \frac{d\hat{q}_2}{dw} \right] = \\
= \left[ (\hat{p}_1 - c_1) + \hat{q}_1 \frac{\partial p_1}{\partial q_1} \frac{d\hat{q}_1}{dw} + \hat{q}_1 \frac{\partial p_1}{\partial q_2} \frac{d\hat{q}_2}{dw} \right] + (\hat{p}_2 - c_2)\frac{d\hat{q}_2}{dw} + \hat{q}_2 \left[ \frac{\partial p_2}{\partial q_1} \frac{d\hat{q}_1}{dw} + \frac{\partial p_2}{\partial q_2} \frac{d\hat{q}_2}{dw} \right] = \\
= \hat{q}_1 \frac{\partial p_1}{\partial q_2} \frac{d\hat{q}_2}{dw} + \hat{q}_2 \frac{\partial p_2}{\partial q_1} \frac{d\hat{q}_1}{dw} = 0
\]

By comparing the last equalities in (6) and (8), it is straightforward that the latter has an additional term,

\[
\frac{\partial p_i}{\partial q_2} \frac{d\hat{q}_2}{dw} > 0,
\]

where the positive sign stems from the fact that \( \frac{\partial p_1}{\partial q_2} < 0 \) and \( \frac{d\hat{q}_2}{dw} < 0 \). Any given increase in the input price will decrease sales of \( D_2 \) which will in turn increase the final-good price and the merged firm's profits from downstream operations. Clearly, the independent upstream firm \( U_2 \) cannot internalize this positive effect and thus the input price will increase...
as a result of the merger. Unlike the pre-merger case, under a general inverse demand function, we cannot determine whether the equilibrium input price will be lower (a subsidy to \(D2\)) or higher than upstream marginal cost. What we can determine, however, is that even if \(D2\) still receives a subsidy post-merger, the amount of that subsidy is lower than the corresponding amount under the pre-merger case.

The effect of the merger on consumer surplus is then clear-cut. Since the equilibrium input price increases as a result of the merger, we know from (3) that both final-good prices will increase. It then follows immediately that consumer surplus must be lower.

**Proposition 1.** A horizontal merger between the vertically integrated firm and the independent upstream supplier will (i) increase the equilibrium input price, \(w^M^* > w^S^*\) and (ii) reduce consumer surplus, \(CS^M^* < CS^S^*\).

Proposition 1 provides support of the basic downstream antitrust concern in such upstream horizontal mergers: a merger between the vertically integrated firm and the independent upstream supplier allows the latter to induce a more accommodating behavior downstream by elevating the input price to the independent downstream firm and thus shifting sales of the final-good to its downstream affiliate. Ultimately, a higher input price leads to higher final-good prices thus making consumers worse off.

Under our modelling structure, in the pre-merger case, all of the vertically integrated firm’s upstream production is directed to its downstream affiliate (captive sales) implying that the merger does not affect concentration in the upstream market: the upstream market is monopolized both in the pre- and post-merger case. Therefore, the merger’s negative impact on consumer surplus stems solely from a vertical partial foreclosure effect. This finding highlights the important role of vertically integrated firms in horizontal merger analysis.

The merger is always beneficial for the merging parties and the industry as a whole. In order to see why the merger increases industry profits, note that:

\[
\pi_{ind}(w^*) = (\hat{p}_1(w^*) - c_1)\hat{q}_1(w^*) + (\hat{p}_2(w^*) - c_2)\hat{q}_2(w^*) \text{ with } k = S, M.
\]

Recall that, in the pre-merger case, the input price is chosen so as to maximize the unintegrated vertical chain’s profits, \((\hat{p}_1(w) - c_1)\hat{q}_1(w)\), and not industry profits. In the post-merger case, however, the input price is chosen so as to maximize overall industry profits, \(\pi_{ind} = (\hat{p}_1(w) - c_1)\hat{q}_1(w) + (\hat{p}_2(w) - c_2)\hat{q}_2(w)\). Therefore, it must hold that \(\pi_{ind}^M^* > \pi_{ind}^S^*\).
Since overall industry profits increase as a result of the merger, and $D2$’s net profits remain unaffected (in both cases are equal to zero), it must hold that the combined net profits of $U1-D1$ and $U2$ increase, implying that the merger is beneficial for the merging parties.

The effect of the merger on total welfare is \textit{a priori} ambiguous since it decreases consumer surplus but increases industry profits. In fact, there are two opposing effects on total welfare as a result of the merger. On the one hand, as we show formally in the Appendix, the merger has a welfare-decreasing effect since it decreases the total quantity supplied in the downstream market, i.e., $d(q_i + \hat{q}_2)/dw < 0$. On the other hand, the merger has a welfare-enhancing effect: since it increases the input price paid by $D2$, sales will be shifted towards the downstream division of the merged firm $I$ which is more cost-efficient increasing allocative efficiency. Under what conditions the latter effect is strong enough so that the upstream merger increases total welfare even though consumer surplus decreases? Unfortunately, we cannot say without imposing more structure on the demand side of the model; this task is undertaken in the next subsection where we consider the case of a linear demand function.

3.3. Linear demand

In order to further illustrate our general findings and investigate the effect of the merger on total welfare, we consider a linear demand example. More specifically, we assume the following inverse demand function,

$$p_i = 1 - q_i - \theta q_j, \quad i, j = 1, 2, \quad i \neq j, \quad 0 \leq \theta < 1, \quad (9)$$

where the parameter $\theta$ measures the degree of product substitutability. The higher is $\theta$, the closer substitutes final goods are. In the limit, as $\theta$ approaches 1, final goods become completely homogeneous. In order to simplify calculations when performing comparisons later, following Arya \textit{et al.} (2008), we introduce here the parameters $a_i = 1 - c_i$ and $a_2 = 1 - c_2$ where $a_i$ is the difference between the intercept of firm $D_i$’s inverse demand function and its downstream marginal production cost. Given that $c_i < c_2$ it holds that $a_i > a_2$.

We also make the following assumption:

**Assumption 1.** $\frac{a_2}{a_i} > \theta$. 

12
Assumption 1 requires that the production cost advantage of \( U1-D1 \) over \( D2 \) is not too high (the ratio \( a_z/a_i \) is relatively high) and/or final-goods are sufficiently differentiated (the parameter \( \theta \) is relatively low). It guarantees that both firms will produce a positive quantity of the final-good under both the pre- and post-merger case(s). As can be seen from (14) below, when \( a_z/a_i \leq \theta \) then \( q_{z}^{u''} \leq 0 \), which implies that \( D2 \) is fully foreclosed from the market. Therefore, Assumption 1 can also be interpreted as a non-foreclosure condition (see also Arya et al., 2008).

As it was already explained in the previous subsections, when \( D2 \) procures the input from \( U2 \), the downstream equilibrium outcomes are the same regardless of whether the upstream merger occurs or not. Using the demand functions in (9), we obtain the last-stage subgame equilibrium outcomes as:

\[
\hat{q}_1(w) = \hat{p}_1(w) - c_1 = \frac{2a_i - \theta a_z + \theta w}{4 - \theta^2}, \quad \hat{q}_2(w) = \hat{p}_2(w) - c_2 = \frac{2a_2 - \theta a_i - 2w}{4 - \theta^2}. \tag{10}
\]

From (6), we obtain the equilibrium input price in the pre-merger case,

\[
w^{s*} = -\frac{\theta^2 (2a_2 - \theta a_i)}{4(2 - \theta^2)}, \tag{11}
\]

Given Assumption 1 it holds that \( w^{s*} < 0 \) which verifies Lemma 1. In the pre-merger case, the equilibrium input price reflects a subsidy from \( U2 \) to its respective downstream firm \( D2 \), and this subsidy is higher, the higher is the degree of substitutability between final goods, \( dw^{s*}/d\theta < 0 \). The closer substitutes final goods are, the stronger is downstream competition, and thus the more valuable is to be aggressive in the final-good market.

The final equilibrium outcomes are given by:
From (9), we obtain the equilibrium input price in the post-merger case,

$$w_{M*}^{**} = \frac{\theta[(4 + \theta^2)\alpha_1 - 4\delta\alpha_2]}{2(4 - 3\theta^2)} > 0.$$  

(13)

It holds that \( w_{M*}^{**} > 0 > w^{**} \) which verifies the first part of Proposition 1. Under this specific demand function under consideration, the equilibrium input price post-merger turns out to be positive implying that \( D2 \) no longer receives a subsidy. It also holds that \( dw_{M*}^{**}/d\theta > 0 \), which implies that the closer substitutes final goods are, the more urgent is for the newly merged firm to induce a more accommodating behavior downstream by elevating the input price to the independent downstream firm and thus shifting sales of the final-good to its downstream affiliate.

The final equilibrium outcomes are given by:

$$q_{1*}^{**} = (p_{1*}^{**} - c_1) = \frac{(4 - \theta^2)\alpha_1 - 2\theta\alpha_2}{2(4 - 3\theta^2)}, \quad q_{2*}^{**} = (p_{2*}^{**} - w_{M*}^{**} - c_2) = \frac{2(\alpha_2 - \theta\alpha_1)}{(4 - 3\theta^2)},$$

$$F_{M*}^{**} = \frac{(4a_2 - \theta\alpha_1)^2}{(4 - 3\theta^2)^2}, \quad \pi_{i*}^{M*} = \frac{a_1^2(4\theta^2 + 8\theta\alpha_2 - 8a_2\theta)}{8(4 - 3\theta^2)}, \quad \pi_{D2}^{M*} = 0,$$

(14)

$$CS^{M*} = \frac{1}{2}(1 - p_{1*}^{M*})q_{1*}^{M*} + \frac{1}{2}(1 - p_{2*}^{M*})q_{2*}^{M*} = \frac{a_1^2(4 - 3\theta^2) + 4a_2^2 - 4a_1\alpha_2\theta}{8(4 - 3\theta^2)},$$

$$TW^{M*} = CS^{M*} + \pi_{i*}^{M*} = \frac{a_1^2(12 - \theta^2) + 12a_2^2 - 20a_1\alpha_2\theta}{8(4 - 3\theta^2)}.$$
Using (12) and (14), we calculate the following expressions:

\[ \pi^M_1 - (\pi^S_{U1-D1} + \pi^S_{U2}) = \frac{A^2\theta^2}{16(4-3\theta^2)(2-\theta^2)^2} > 0 \]

\[ CS^M - CS^S = \frac{\theta AB}{32(4-3\theta^2)(2-\theta^2)^2} < 0 \quad \text{(15)} \]

\[ TW^M - TW^S = \frac{\theta AC}{32(4-3\theta^2)(2-\theta^2)^2} < 0 \]

where \( A = (4 - \theta^2)a_1 - 2\theta a_2, \quad B = (4 - 3\theta^2)a_1 - 2a_2(8 - 5\theta^2) \) and \( C = (12 - 5\theta^2)a_1 - 2a_2(8 - 3\theta^2) \).

For \( 0 \leq \theta < 1 \) and \( a_1 > a_2 > a_1\theta \) it holds that \( A > 0, \quad B < 0 \) and \( C < 0 \). Therefore, the merger increases industry profits (and consequently is beneficial for the merging parties) and decreases both consumer surplus and total welfare. As it turns out, at least for the case of a linear demand, the resulting increase in allocative efficiency due to the merger is not enough to compensate for the fact that total output is reduced and thus total welfare is lower compared to the pre-merger case.

4. The modified model with an upstream competitive fringe

In this section, we assume that the independent upstream firm \( U_2 \) (or the merged firm \( I \) in case the merger occurs) faces a fringe of equally efficient rivals. Competition among the fringe suppliers ensures that the input price will be equal to the fringe’s marginal cost if \( D_2 \) turns to the fringe for its supply. To ensure tractability, we continue to assume that consumers’ demand is linear and given by (9). Clearly, when \( D_2 \) procures the input from the competitive fringe, the final equilibrium outcomes are given by (10) with \( w^{CF} = 0 \), i.e.,

\[ q_1^{CF} = p_1^{CF} - c_1 = \frac{2a_1 - \theta a_2}{4 - \theta^2}, \quad q_2^{CF} = p_2^{CF} - c_2 = \frac{2a_2 - \theta a_1}{4 - \theta^2}. \quad \text{(16)} \]

where the superscript \( CF \) is used to denote equilibrium outcomes when \( D_2 \) resorts to the competitive fringe.

4.1. The pre-merger case

15
Let us look first what happens in the pre-merger case. The independent upstream firm $U_2$ chooses $w$ and $F$ to maximize,

$$\max_{w,F} w \hat{q}_2(w) + F,$$

subject to

$$(\hat{p}_2(w) - w - c_2)\hat{q}_2(w) - F \geq \pi_{D2}^{CF*}. \quad (18)$$

The inequality in (18) is $D_2$’s incentive compatibility constraint - the constraint that $D_2$ does not resort to the alternative supply. Upon substituting (18), which obviously holds as an equality, into (17), the input price is set so as to maximize,

$$\max_w (\hat{p}_2(w) - c_2)\hat{q}_2(w) - \pi_{D2}^{CF*}. \quad (19)$$

Since the choice of $w$ does not affect $\pi_{D2}^{CF*}$, the above maximization problem is essentially the same as in the case without an alternative supply (see (5)). Therefore, the equilibrium input price will be still given by (11). Since $U_2$ can offer better terms to $D_2$ than the competitive fringe (recall that the input price reflects a subsidy) then the final equilibrium prices, outputs, vertically integrated chain’s profits, consumer surplus and total welfare will be the same as in the absence of the competitive fringe (see (12)). However, the distribution of profits between $U_2$ and $D_2$ will not be the same: $D_2$ now earns $\pi_{D2}^{CF*}$ and thus $U_2$ can make only $\pi_{U2}^{CF*} - \pi_{D2}^{CF*}$.

4.2. The post-merger case

We now turn to the post-merger case. The merged firm $I$ chooses $w$ and $F$ to maximize,

$$\max_{w,F} (\hat{p}_1(w) - c_1)\hat{q}_1(w) + w \hat{q}_2(w) + F$$

subject to the constraint in (18). Upon substituting (18), which holds as an equality, into (20), the input price is set so as to maximize,

$$\max_w (\hat{p}_1(w) - c_1)\hat{q}_1(w) + (\hat{p}_2(w) - c_2)\hat{q}_2(w) - \pi_{D2}^{CF*}. \quad (21)$$
Similar to the pre-merger case, since the choice of \( w \) does not affect \( \pi_{D2}^{CF} \), the maximization problem in (21) is essentially the same as in the case without an alternative supply (see (7)). The input price that maximizes overall industry profits is still given by (13). However, at that input price, \( D2 \) makes higher gross profits when procures the input from the competitive fringe (recall that \( w^{M*} > w^{CF} = 0 \)), which implies that firm \( I \) must offer a negative fixed fee to \( D2 \) in order to compensate the latter for the higher input price. Therefore, firm \( I \) is constrained by the presence of the competitive fringe: at best, it can offer precisely the same conditions as the fringe to \( D2 \), that is \( \bar{w}^{M*} = w^{CF} = 0 \) and a zero fixed fee.

The final equilibrium outcomes are given by:

\[
\bar{q}_1^{M*} = (\bar{p}_1^{M*} - c_1) = \frac{2a_1 - \theta a_1}{4 - \theta^2}, \quad \bar{q}_2^{M*} = (\bar{p}_2^{M*} - c_2) = \frac{2a_2 - \theta a_1}{4 - \theta^2},
\]

\[
\bar{T}_I^{M*} = \frac{(2a_1 - \theta a_1)^2}{(4 - \theta^2)^2}, \quad \bar{T}_{D2}^{M*} = \frac{(2a_2 - \theta a_1)^2}{(4 - \theta^2)^2},
\]

\[
\bar{CS}^{M*} = \frac{(a_1^2 + a_2^2)(4 - 3\theta^2) + 2a_1a_2\theta^3}{2(4 - \theta^2)^2},
\]

\[
\bar{TW}^{M*} = \frac{(a_1^2 + a_2^2)(12 - \theta^2) - 2a_1a_2\theta(8 - \theta^2)}{2(4 - \theta^2)^2}.
\]

Before turning to comparisons between the pre- and post-merger cases, we now relax Assumption 1. As it can be easily seen from the equilibrium outputs in (12) and (22), the assumption that \( 2a_2 - \theta a_1 > 0 \) guarantees that both firms will produce a positive quantity of the final-good under both the pre- and post-merger case(s). Therefore, in the presence of an upstream competitive fringe, we make the following assumption:

**Assumption 2.** \( \frac{a_2}{a_1} > \frac{\theta}{2} \)

Note that Assumption 2 is less restrictive than Assumption 1. This is straightforward since the possibility of full foreclosure, which existed in the post-merger case in the absence of the competitive fringe, does not longer exist due to the presence of an alternative source of supply. Given Assumption 2, we obtain the following Proposition.
Proposition 2. In the presence of an upstream competitive fringe and under a linear demand function, a horizontal merger between the vertically integrated firm and the independent upstream supplier: (i) is always profitable, (ii) will always increase the equilibrium input price and reduce consumer surplus, (iii) will increase total welfare if and only if the following condition holds,

\[
\frac{a_2}{a_1} < \frac{\theta(64 - 36\theta^2 + 3\theta^4)}{2(32 - 12\theta^2 - \theta^4)}.
\]

Proof. We calculate the following expressions:

(i) \(\bar{\pi}_I^{M^*} - (\bar{\pi}_{U-D}^{s^*} + \pi_{U2}^{s^*} - \pi_{D2}^{cF^*}) = \frac{\theta^2 (2a_2 - \theta a_1)[a_1(32 - 12\theta^2 - \theta^4) - 2\theta a_2(12 - 5\theta^2)]}{16(8 - 6\theta^2 + \theta^4)}\).

The above expression has the sign of the bracketed term, which is positive whenever \(\frac{a_2}{a_1} < \frac{(32 - 12\theta^2 - \theta^4)}{2\theta(12 - 5\theta^2)}\) holds. The latter inequality is always true given that \(a_2 < 1\) and \(\frac{(32 - 12\theta^2 - \theta^4)}{2\theta(12 - 5\theta^2)} > 1\).

(ii) \(\bar{w}_{M^*} - w_{s^*} = \frac{\theta^2 (2a_2 - \theta a_1)}{4(2 - \theta^2)} > 0\),

\(\bar{cS}_{M^*} - CS_{s^*} = -\frac{\theta^2 (2a_2 - \theta a_1)[a_1\theta^3(12 - 5\theta^2) + 2a_2(32 - 36\theta^2 + 9\theta^4)]}{32(8 - 6\theta^2 + \theta^4)} < 0\).

(iii) \(\bar{TW}_{M^*} - TW_{s^*} = \theta^2 (2a_2 - \theta a_1)[a_1\theta(64 - 36\theta^2 + 3\theta^4) - 2\theta a_2(32 - 12\theta^2 - \theta^4)]\).

The last expression has the sign of the bracketed term, which is positive whenever \(\frac{a_2}{a_1} < \frac{\theta(64 - 36\theta^2 + 3\theta^4)}{2(32 - 12\theta^2 - \theta^4)}\) holds. ■

In the pre-merger case, the independent upstream firm is unconstrained by the presence of an alternative source of supply as it can offer better terms to the independent downstream firm. However, in the post-merger case, the independent downstream firm’s threat of switching to the fringe limits the market power of the merged firm; the merger will still increase the equilibrium input price and decrease consumer surplus but not to the same extent as in the absence of the fringe. More importantly, the presence of the competitive fringe introduces the possibility that the merger will increase total welfare even though consumer
surplus decrease. As Proposition 2 indicates, the merger is welfare-enhancing when final goods are close enough substitutes and/or the cost advantage of the vertically integrated chain vis-à-vis the independent downstream firm is sufficiently high (see Fig. 3 below). This finding is important since it implies that whether antitrust authorities favor a consumer or total welfare objective can lead to very different conclusions regarding the merger’s desirability.

\[
\frac{a_2}{a_1} = \frac{\theta (64 - 36\theta^2 + 3\theta^4)}{2(32 - 12\theta^2 - \theta^4)}
\]

Figure 3. In the presence of an upstream competitive fringe, the shaded area denotes the values of \(a_2/a_1\) and \(\theta\) under which the merger increases total welfare.

5. Conclusions

In this paper, we have studied upstream horizontal mergers when one of the merging parties is a vertically integrated firm. We have considered a two-tier market consisting of two competing vertical chains, with one upstream and one downstream firm in each chain. We
have assumed that one vertical chain is vertically integrated whereas the other chain is vertically separated. We have also assumed that the vertically integrated chain is more cost-efficient in its downstream operations than the independent downstream firm.

With downstream Cournot competition, and under a general demand function, we have shown that a horizontal merger between the vertically integrated firm and the independent upstream supplier allows the latter to induce a more accommodating behavior downstream by elevating the input price paid by the independent downstream firm and thus shifting sales of the final-good to its downstream affiliate. Ultimately, a higher input price leads to higher final-good prices thus making consumers worse off. Under our modelling structure, in the pre-merger case, all of the vertically integrated firm’s upstream production is directed to its downstream affiliate (captive sales) implying that the merger does not affect concentration in the upstream market: the upstream market is monopolized both in the pre- and post-merger case(s). Therefore, the merger’s negative impact on consumer surplus stems solely from a vertical partial foreclosure effect. This finding highlights the important role of vertically integrated firms in horizontal merger analysis.

We have shown that the merger is always beneficial for the merging parties and the industry as a whole. Therefore, the effect of the merger on total welfare is a priori ambiguous since it decreases consumer surplus but increases industry profits. On the one hand, the merger decreases total welfare since it decreases the total quantity supplied in the downstream market. On the other hand, the merger has also a welfare-enhancing effect: since it increases the equilibrium input price, sales will be shifted towards the downstream division of the newly merged firm which is more cost-efficient thereby increasing allocative efficiency. By employing a linear demand function, we have shown that the resulting increase in allocative efficiency due to the merger is not enough to compensate for the fact that total output is reduced and thus total welfare is lower compared to the pre-merger case.

We have also investigated the welfare effects of the merger when an upstream competitive fringe exists while still restricting attention, for tractability reasons, to a linear demand function. In the pre-merger case, the independent upstream firm is essentially unconstrained by the presence of an alternative source of supply as it can offer better terms to the independent downstream firm. However, in the post-merger case, the independent downstream firm’s threat of switching to the fringe limits the market power of the merged firm; the merger will still increase the equilibrium input price and decrease consumer surplus but not to the same extent as in the absence of the fringe. More importantly, the presence of
the competitive fringe introduces the possibility that the merger will increase total welfare even though consumer surplus decrease. More specifically, it has been shown that the merger is welfare-enhancing when final goods are close enough substitutes and/or the cost advantage of the vertically integrated chain vis-à-vis the independent downstream firm is sufficiently high. This finding is important since it implies that whether antitrust authorities favor a consumer or total welfare objective can lead to very different conclusions regarding the merger’s desirability.

Appendix

The last-stage subgame equilibrium final-good outputs and prices as functions of the input price are given by: \( \hat{q}_1(w) \), \( \hat{q}_2(w) \), \( \hat{p}_1(w) = p_1[\hat{q}_1(w), \hat{q}_2(w)] \) and \( \hat{p}_2(w) = p_2[\hat{q}_1(w), \hat{q}_2(w)] \). As noted in subsection 2.2, these equilibrium outcomes are the same regardless of whether the merger occurs or not. We show that:

\[
\frac{d\hat{q}_1(w)}{dw} > 0, \quad \frac{d\hat{q}_2(w)}{dw} < 0, \quad \frac{d\hat{p}_1(w)}{dw} > 0, \quad \frac{d\hat{p}_2(w)}{dw} > 0.
\]

Note first that \( \hat{q}_l \) depends on \( w \) only indirectly through \( \hat{q}_2 \) so that \( \hat{q}_l(w) = q_l[\hat{q}_2(w)] \) and

\[
\frac{d\hat{q}_l(w)}{dw} = \frac{dq_l}{dq_2} \frac{dq_2}{dw}.
\]

Given strategic substitutability (see Assumption 2) it holds that \( \frac{dq_1}{dq_2} < 0 \). It is then straightforward that \( \frac{d\hat{q}_l(w)}{dw} \) and \( \frac{d\hat{q}_2(w)}{dw} \) have opposite signs. We next show that \( \frac{d\hat{q}_2(w)}{dw} < 0 \).

The last-stage subgame equilibrium final-good outputs \( \hat{q}_1(w) \) and \( \hat{q}_2(w) \) must satisfy the first-order conditions in the downstream market, therefore (2) can be written as:

\[
p_2[\hat{q}_1(w), \hat{q}_2(w)] + \hat{q}_2(w) \frac{\partial p_2}{\partial q_2} - w - c_2 = 0.
\]

Using the implicit function theorem in the above expression, we obtain:

\[
\frac{d\hat{q}_2(w)}{dw} = \frac{1}{2 \frac{\partial^2 \pi_{p2}}{\partial q_2^2} - \frac{\partial^2 p_2}{\partial q_2 \partial q_1} \frac{dq_1}{dq_2} < 0},
\]
where the denominator \( \frac{\partial^2 \pi_{D2}}{\partial q_{D2}^2} \) is negative due to Assumption 1. Therefore, it holds that

\[
\frac{d\hat{q}_2(w)}{dw} < 0 \quad \text{and} \quad \frac{d\hat{q}_1(w)}{dw} > 0.
\]

Moreover, given that \( \left| \frac{dq_1}{dq_2} \right| < 1 \), it also holds that

\[
\frac{d\hat{q}_1(w)}{dw} < \frac{d\hat{q}_2(w)}{dw}.
\]

The last inequality implies that an increase in the input price decreases the total quantity supplied in the downstream market, i.e., \( \frac{d(q_1 + \hat{q}_2)}{dw} < 0 \).

Regarding the effect of \( w \) on \( \hat{p}_2 \), we have that,

\[
\frac{d\hat{p}_2(w)}{dw} = \frac{\partial \hat{p}_2}{\partial q_2} \frac{d\hat{q}_2(w)}{dw} + \frac{\partial \hat{p}_2}{\partial q_1} \frac{d\hat{q}_1(w)}{dw}
\]

\[
= \frac{\partial \hat{p}_2}{\partial q_2} \frac{d\hat{q}_2(w)}{dw} + \frac{\partial \hat{p}_2}{\partial q_1} \frac{dq_1}{dq_2} \frac{d\hat{q}_2(w)}{dw}
\]

\[
= \frac{d\hat{q}_2(w)}{dw} \left[ \frac{\partial \hat{p}_2}{\partial q_2} + \frac{\partial \hat{p}_2}{\partial q_1} \frac{dq_1}{dq_2} \right] > 0,
\]

where the bracketed term in the last inequality is negative since \( \left| \frac{\partial \hat{p}_2}{\partial q_2} > \frac{\partial \hat{p}_2}{\partial q_1} \right| \) and \( \frac{dq_1}{dq_2} < 1 \).

Finally, regarding the effect of \( w \) on \( \hat{p}_1 \), we have that,

\[
\frac{d\hat{p}_1(w)}{dw} = \frac{d\hat{q}_2(w)}{dw} \left[ \frac{\partial \hat{p}_1}{\partial q_2} + \frac{\partial \hat{p}_1}{\partial q_1} \frac{dq_1}{dq_2} \right] > 0.
\]

An increase in \( w \) affects indirectly \( \hat{p}_1 \) through \( \hat{q}_2 \) in two ways: On the one hand, a decrease in \( \hat{q}_2 \) increases \( \hat{p}_1 \) - a second order effect. On the other hand, a decrease in \( \hat{q}_2 \) leads to an increase in \( \hat{q}_1 \) which in turn decreases \( \hat{p}_1 \) - a third order effect. Naturally, the second order effect is of greater importance than the third order effect implying that \( \hat{p}_1 \) increases with \( w \). ■
References