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Abstract

This paper studies dynamic competition in a duopoly under the assumption that prices are sticky, that is, they do not adjust instantaneously to the level implied by the quantities produced by the two firms. Assuming that market demand is static, contrary to the traditional approach according to which demand evolves dynamically following the course of prices, the equilibrium prices are conjectured to be higher than the Cournot level since at that level the marginal direct benefit of a quantity increase is strictly lower than the marginal indirect cost of a future price reduction. Therefore, sticky prices have an effect similar to that of the fear of price wars that keeps prices high.

JEL classification: L11; L12; L13; L22; L42;
Keywords: dynamic price competition, sticky prices.

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1 Introduction

An interesting question concerning price formation relates to why, in several cases, prices are sticky. Among the answers that have been proposed are informal collusion and the kinked demand curve model. The former is based on a dynamic game theoretic model, the second on a static one. According to the implicit collusion story firms informally agree to keep prices high in fear of triggering price wars. In the kinked demand curve tradition, a firm faces a highly elastic demand for price increases, since rival firms will not follow a price increase, and an inelastic demand for price decreases since rivals will match price cuts. Technically, this implies that the marginal revenue exhibits a jump at the equilibrium price and, hence, the equilibrium price remains unchanged even after marginal cost changes.

In the present study we build a dynamic model that is based on the fundamental assumption of the kinked demand curve and the sticky price model as formalized by Fershtman and Kamien (1987). The model is based on the idea that, for some reasons, the price does not adjust instantaneously to the level implied by the currently produced quantities. In particular, the evolution of price is assumed to be described by a linear differential equation, which firms take into account in order to maximize their profits by choosing optimally their quantities. The firms are assumed to produce homogeneous products. Importantly, contrary to the assumption of Fershtman and Kamien (1987), that the firms can sell whatever quantity they wish at the current price level, which goes hand in hand with the Cournot model, we assume that this is not possible. Since market price does not instantaneously change to equate demand and supply we have to adopt certain demand rules taking also into account the fact that firms homogeneous products. The simplest of such rules is based on the kinked demand curve approach. However, even under this assumption regarding individual firm demands, our approach, which is perhaps more plausible in the current context than that of Fershtman and Kamien (1987), comes at a huge cost in terms of tractability. Therefore, we restrict our analysis to the study of a static version and provide a set of conjectures regarding the equilibrium of the dynamic model. The most important of these conjectures is that the fully collusive, monopoly price under fairly general conditions can be supported due to price stickiness, without relying on trigger strategies and threats for price wars.

The rest of the paper is organized as follows. Section 2 contains a brief discussion of the related literature. Section 3 presents the model and section 4 describes the equilibrium of the static version of the model. Section 5 presents the basic result of the paper, the conjecture regarding equilibrium prices of the dynamic model. Section 6 discusses some extensions and section 7 concludes.
2 Literature review

Part of the literature on dynamic pricing has been built around the hypothesis of sticky prices. The sticky price model, formalized by Fershtman and Kamien (1987) (FK hereafter) as a duopoly game, is a dynamic model that has found applications in areas such as industrial organization and international trade.\(^1\) The model is built on the assumption that the price does equilibrate current supply with the long-run demand. Price evolves over time and its dynamics are governed by a linear differential equation. The firms choose controls to maximize the present value of a profit stream discounted by a common rate.

To gain a better understanding of the sticky price model, one has to trace its history, which goes back to Roos (1925, 1927). The basic assumption is that consumers’ demand is a function of the price, \(p(t)\), and the rate of change of price, \(dp(t)/dt\). Roos studied the dynamic problem by applying the methods of the calculus of variations. Later, with the establishment of dynamic programming and optimal control techniques in economic modelling, the problem was recast as a differential game. Simaan and Takayama (1978) solved the differential game assuming, additionally, that firms operate under capacity constraints. Until then, the dynamic game was not described as a sticky price model. This term was introduced by FK, that provided a detailed analysis of the problem posed by Roos.

The assumption that drives the results of FK, and in large part the results of related studies that ensued, concerns consumer demand; it is assumed that current willingness to pay for the product depends on current prices and on past consumption (see also discussion in Dockner and Haug, 1990), which is consistent with a utility function that depends on current and past levels of consumption. This is captured by the assumption that the price (i.e. the inverse demand function) evolves as a differential equation. In turn, current demand may be higher compared to its static counterpart. Given that firms face a higher demand, at least in the beginning of the game, they tend to produce more quantity than in the static Cournot model, which results in an equilibrium price that is lower than in the static game. The striking result of FK is that, as the speed of adjustment, goes to infinity and firms use feedback Nash strategies, the equilibrium price reaches a level which is lower than the static Cournot equilibrium price. In this sense, their model does not converge to the static model.\(^2\) This basic result of FK was more or less verified in a number of extensions. Cellini and Lambertini (2004) and Dockner (1988) study the \(N\)-firm

\(^1\)For an extensive discussion on the sticky price model see e.g. Van Long (2010).

\(^2\)This holds if firms use feedback strategies. In the open-loop case there is convergence but the open-loop strategies are generally considered inappropriate for the study of behavior in contexts of (dynamic) interaction.
oligopoly case. Wiszniewska-Matyszkiel et al. (2014) provide a comprehensive analysis of FK’s model for the oligopoly case focusing, in particular, on firms’ behaviour off the steady-state price path. Benchekroun (2003) and Dockner and Gaunersdorfer (2001) use the model of FK to analyse the profitability of horizontal mergers. Cellini and Lambertini (2007) assume that firms sell differentiated products, Piga (2000) shows that advertising reduces the incentive of firms to expand output, still the price in the feedback equilibrium is lower than the Cournot price. Similar results are obtained in more structural approaches like that of Driskill and McCafferty (1989), where it is assumed that firms face adjustment costs when they change the rate of production. It is found that the subgame perfect equilibrium price lies between the Cournot and the perfectly competitive price. An exception to this literature is provided by Tsutsui and Mino (1990) who provide an analysis of nonlinear feedback strategies to the setup of FK and find that a continuum of equilibria arises under the existence of price ceilings, which may be imposed by government intervention.

The prediction of FK and related studies, that as the speed of adjustment tends to infinity the feedback Nash equilibrium tends to a price lower than the Cournot, is generally considered counterintuitive since, price stickiness results in lower prices whereas, normally, the opposite could be expected. Evidence from markets characterized by dynamic and sluggish evolution of demand and/or supply e.g. the oil market, seems not to be in line with the predictions of the sticky price model. Wirl (2010) provides an oligopoly model of sluggish demand and supply showing that the equilibrium price can be higher than the static Cournot price.

3 The model

In this paper we provide a way of modelling sticky prices that differs from FK only regarding current demand. Thus, we assume that total demand is linear in the market price, \( D(p) = a - p \). Importantly, however, we assume that current demand is static, that is, it depends only on current prices and not on past consumption. This assumption implies that, if the current price \( p \) is relatively high, firms might wish to sell a total quantity that exceed \( D(p) \). In order to deal with this event, we assume that a firm cannot sell more than its rival if both firms wish to serve total demand, \( D(p) \). Therefore, we assume that firm \( i \) \((= 1, 2)\) sells:

\[
D_i(p, u_i, u_j) := \min\{u_i, \max\{D(p) - u_j, D(p)/2\}\},
\]

where \( u_k \) denotes the production of firm \( k \) \((i \text{ or } j)\). Function \( \max\{D(p) - u_j, D(p)/2\} \) represents demand for firm \( i \)'s product. The above specification reflects the idea that
a firm cannot sell more than what it produces, but also cannot sell more than its rival if \( u_1, u_2 \geq D(p)/2 \), that is, if total production is enough to serve total demand total demand is split equally among the two firms. In the case where \( u_j < D(p)/2 \), firm \( i \) could sell up to the residual demand, \( D(p) - u_j > D(p)/2 \).

Since our focus is on price equilibrium when firms cannot change their prices instantly, contrary to the Cournot assumption of instant price adjustment to quantities, we assume that the price responds gradually to changes in quantities. If a firm increases its supply the price would not drop instantly to clear the market. This could happen either because of menu costs, or because consumer demand does not respond instantly to a price drop, not because past consumption affects demand, but perhaps because of imperfect price information. In short, the current price at period \( t \) does not adjust instantaneously to the level implied by current production but, instead, follows a trajectory described by the following differential equation \( dp/dt = s(a - p - u_1 - u_2) \), which reflects the fact that the price will be increasing if current demand \( D(p) = a - p \) exceeds current production, \( u_1 + u_2 \). The speed of price adjustment depends on parameter \( s \).

The objective of firm \( i \) is to maximize

\[
J_i(p_0, u_i, u_j) := \int_0^\infty e^{-rt}[p(t)D_i(p(t), u_i(p(t)), u_j(p(t))) - cu_i(p(t)) - \frac{1}{2}u_i^2(p(t))]dt \tag{2}
\]

subject to

\[
\frac{dp(t)}{dt} = s[a - p(t) - u_1(t) - u_2(t)],
\]

\( p(0) = p_0 \) and \( u_i(t) \geq 0 \).

The assumption we use regarding firm level demand comes at a huge cost in terms of tractability, which is due to the following fact: If the price is relatively high and, given price stickiness, firms would wish to be able to sell a quantity exceeding the quantity demanded at that (currently high) price level (an issue that does not arise in the model of FK due to the particular inverse demand that is used). In an equilibrium where the firms produce less than \( D(p)/2 \) and the price is rising, no firm should be able gain from producing more than \( D(p)/2 \) when its rival produces less than \( D(p)/2 \).

To make the model tractable, we adopt the following rule regarding individual firm's

\[\text{This issue concerns also the model of FK but, since, demand depends on past consumption as well, and, given that the inverse demand function at } t = 0, \text{ is } p(0) = a, \text{ the related constrained is not binding.}\]

\[\text{At this point an important remark is in order. In our model current price does not depend on current productions. The idea is that, producing more or less that current demand, would decrease or increase future prices. Assuming that current production exceeds current demand, the price would be decreasing according to the above specification but there is an issue with the surplus that cannot be sold in the current period. In the present model we make an additional simplifying assumption: the product is non-storable. We return to this issue in Section 6.}\]
demand functions.

\[ D_i(p, u_i, u_j) := \min\{u_i, D(p)/2\}. \]  

This rule treats the firms symmetrically, in accordance with the assumption of homogeneous products. Importantly, this demand rule approximates the kinked demand curve. More specifically, it is an extreme case of the kinked demand curve since it is implied that, if a firm reduces its price within a period (in order to sell a greater quantity) the other firm will respond immediately. This rule can also be considered to be a specific case of the more general rule that is used in Operations Research, according to which firm demand consists of primary and secondary demand as fixed proportions of total demand and total non served demand, respectively, as follows:

\[ \xi_i(p, u_i, u_j) := \max\{\lambda_i D(p), \lambda_i D(p) + \gamma_{ji}(\lambda_j D(p) - u_j)\}, \]  

s.t. \( \lambda_1 + \lambda_2 = 1 \). Therefore, the demand function (3) that we employ corresponds to the particular case of (4) where \( \lambda_i = 1/2 \) and \( \gamma_{ji} = 0 \).\(^5\)

Firms can be assumed to employ a variety of strategies e.g. open-loop, feedback or closed-loop.\(^6\) Because feedback strategies are compatible with subgame perfect equilibria, we will assume that firms do follow feedback strategies with the objective to maximize (2). Importantly, unlike FK, the present model is not linear-quadratic. Moreover, the firms’ objective functions are not everywhere differentiable with respect the state and the control variable. The value function is not quadratic and Markov strategies are nonlinear. In short, this implies that our model cannot be solved using the traditional methods of smooth analysis.\(^7\) But as in FK, current production does affect only future price, not the current price. The technical complications that arise certainly prevent us from deriving a complete set of results. We thus resort to a conjecture which, under the assumptions above, seems quite plausible. Before this analysis, in the next section we provide a short presentation of the much simpler static version of our model. This analysis reveals insights which, in part, carry over in the dynamic version.

\(^5\)Parameter \( \gamma_{ji} \) is the proportion of demand that is not served by firm \( j \) which can be served by firm \( i \). Thus, \( \gamma_{ji} \in [0, 1] \). Note, also, that \( D_i(p, u_i, u_j) := \min\{u_i, \xi_i(p, u_i, u_j)\} \).

\(^6\)For more on this subject see e.g. Basar and Olsder (1999).

\(^7\)For a first reading in the techniques of nonsmooth analysis see e.g. Clarke (1983) and Vinter (2000).
4 Equilibrium in the static version

In the static version of our model the two firms live for a single period and the price is fixed at level $p$. Because dynamics are absent, this model is quite simple and can be built on assumption (1). The two firms choose simultaneously and noncooperatively their quantities with the objective to maximize their profits. The profit of firm $i$ is $J_i(u_1, u_2) = pD_i(p, u_1, u_2) - cu_i - \frac{1}{2}u_i^2$. The equilibrium of this model is described by the following:

**Proposition 1** If the two firms live for a single period and sell at a certain price, $p$, the equilibrium quantities are

$$u^*_1 = u^*_2 = \begin{cases} p - c & \text{if } p - c \leq D(p)/2 \\ D(p)/2 & \text{otherwise.} \end{cases}$$

This first static view of our model leads to a simple intuitive result. If the price is independent of the quantities produced, the firms would not produce more than the quantity demanded at price $p$. We note also that firms produce more as the price goes up, a prediction verified by FK, but only if the price does not exceed a certain level; if $p$ is higher than the competitive price, $\frac{1}{3}(a + 2c)$, the demand constraint becomes binding, firms’ total production equals demand, hence, total production is negatively related to the market price. Note also that the above equilibrium can be interpreted as the equilibrium of the dynamic model if the firms are myopic, that is, in every period $t$ each firm has the objective to maximize $[p(t)D_i(p(t), u_i(t), u_j(t)) - cu_i(t) - \frac{1}{2}u_i^2(t)]$.

The important insight gained from this approach concerns the dynamic model. Let the (sticky) price at period $t$ be lower than the optimal collusive price (the monopoly price) and fix production rates at $u_1(t) + u_2(t) = D(p(t))$.\(^8\) Then producing more than $u_i(t)$ would result in lower profit for firm $i$ compared to $u_i(t)$. Clearly, sticky prices reduce the incentive of firms to overflow the market, since this would result to lower future price, no additional revenue and higher production cost. This is the logic behind the conjecture regarding the feedback equilibrium of the differential game, which is presented below.

\(^8\)The collusive price maximizes the joint profit, $\{p(u_1 + u_2) - c(u_1 + u_2) - \frac{1}{2}(u_1^2 + u_2^2)\}$, where $u_i = D(p)/2$. This price is $(3a + 2c)/5$ and differs from that of the FK model because total demand in our mode is constant per period. In the setup of FD this price is given in Tsutsui Mino (1990, p.154). Straightforward calculation shows that the joint profit is a concave function of price.
5 Conjectures on the dynamic model

Before stating our main conjecture regarding let us summarize the differential game we will study below and the equilibrium notion. We assume that the two firms play a differential game of infinite horizon. The objective of firm $i$ is to maximize

$$J_i(p_0, u_i, u_j) := \int_0^\infty e^{-rt}[p(t) \min\{u_i(t), D(p(t))/2\} - cu_i(t) - \frac{1}{2}u_i^2(t)]dt$$

subject to

$$\frac{dp(t)}{dt} = s[a - p(t) - u_1(t) - u_2(t)],$$

$p(0) = p_0$ and $u_i(t) \geq 0$. We focus on feedback Nash strategies, which depend on the state variable, the market price.

Given that the price does not respond instantly to quantity choices, when choosing its current quantity, each firm has to consider its quantity’s effect on the current and future price. Since current price is fixed, it only remains to consider the effect on the future price. Consider that the price is at the collusive level and suppose that the firms split the market in half. If one of the two firms increases its quantity, it will only reduce future price but it will not increase current profit, since the current price cannot drop in order to accommodate the additional quantity. It follows that the collusive price is an equilibrium price. Note that this argument holds also under the more general assumption (1).

The previous argument can be used to extend our intuition if the market price differs from the collusive level. Below we describe three cases, which we illustrate in Figure 1:

(i) Initial price above the collusive price: in this case the two firms have an incentive to reduce the price. This can be done only if at least produces more than $D(p)/2$. The price is expected to fall.

(ii) Initial price below the collusive price but above the critical price $\tilde{p}(r, s)$: the firms would like the price to increase but they are not willing to take the cost from producing less than $D(p)/2$. On the other hand, they do not have an incentive to reduce the price by producing more than $D(p)/2$. It follows that the price remains at its initial level for ever. A continuum of equilibria, each corresponding to initial prices (that remain there for ever) arises. $\tilde{p}(r, s)$ is a decreasing function of $r$ because the more impatient the firms are the less is their willingness to reduce current quantities in order to increase future price and profits. The effect of $s$ on $\tilde{p}(r, s)$ is ambiguous since a higher speed of adjustment would approach the Cournot model (in the limit) but also as $s$ increases it becomes easier to increase the price by producing less, which tends to increase $\tilde{p}(r, s)$. 

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(iii) Initial price below $\tilde{p}(r, s)$. In this case the cost associated with a price increase is lower than the cost and, therefore, the price rises to $\hat{p}(r, s)$. Importantly, $\hat{p}(r, s)$ exceeds the Cournot price.\(^9\) To see this consider that the initial price is at the Cournot level. Absent the gains from a future price increase, each firm would choose to produce just $D(p_{\text{Cournot}})/2$. In the presence of such future gains each firm will produce a strictly lower quantity than the former and the price would increase. It is at this point that assumption (3) is necessary. To see this, note that under assumption (1), if a firm chose to produce less than $D(p_{\text{Cournot}})/2$, its rival would possibly have an incentive to produce more than this and gain more, while the price could also increase (if total quantity would not exceed $D(p_{\text{Cournot}})$). This possibility reduces the chances of observing a price rising significantly above the Cournot level. Finally, note that if $p_0$ is below a lower bound, $\hat{p}$, price dynamics are described by FK (Theorem 2), in which case the optimal controls are $u^*_i = 0$.

We summarize the above discussion in the following:

**Conjectures**

(C1) No firm has an incentive to produce more than half of total demand and reduce the price if the price is below the collusive level.

(C2) Starting from a low level ($\hat{p}(r, s)$), the price converges to $\hat{p}(r, s)$.

\(^9\)The Cournot and competitive prices are $(a + c)/2$ and $(a + 2c)/3$, respectively. For a discussion and derivation see e.g. FK.
(C3) A continuum of steady states with measure that is increasing in \( r \) arises if the (initial) price falls between \( \bar{p}(r, s) \) and the collusive price.

Regarding the collusive price, our conjecture can be further extended to the more general demand function (1). The reason is that because the current price does not instantaneously respond to quantity changes, no firm has an incentive to produce more, since that would only cause a future price decrease and – not an increase of the quantity sold.

6 Extensions

6.1 Oligopoly

The results of the preceding analysis remain qualitatively unchanged under assumption (1) properly modified, to the oligopoly case with \( n \) symmetric firms in the market. Thus, if we assume that firm \( i \)'s demand function is

\[
D_i(p, u_i, u_j) := \min\{u_i, \max\{D(p) - \sum_{j \neq i} u_j, D(p)/n\}\},
\]

the collusive price \([(a + n(a + c))/(1 + 2n)]\) is an equilibrium price. Note, also, that this price is decreasing in the number of firms, which follows from the quadratic production cost function. Under the more restrictive demand function

\[
D_i(p, u_i, u_j) := \min\{u_i, D(p)/n\},
\]

our three conjecture are also expected to hold, since the arguments remain unchanged.

6.2 Storable goods

The reason behind price changes, in our model, is demand or supply surpluses. Price changes are slow, but they exist if the firms decide not to match current demand. So far we have assumed that the product is non storable. Thus, if the production at period \( t \), when the price is \( p_t \), exceeds demand, \( D(p_t) \), the quantity that remains unsold at the end of the day, \( \max\{0, u_1 + u_2 - D(p)\} \), is lost. Assuming, instead, that the product can be stored, implies the presence of inventories. The introduction of inventories complicates dramatically the present model. For one thing, this would require considering a discrete-time model since the presence of inventory over time (which, technically, is formed continuously, as an integral) is of different order of
magnitude than (instant) quantity. The interpretation of the state equation has to
be modified, too, in order to reflect the effect of inventories on the intensity of price
changes. A possible state equation that can be used is the following:

\[ p(t + 1) - p(t) = s[a - p(t) - u_1(t) - u_2(t) - s_1(t) - s_2(t)], \]

where \( s_i(t) \) denotes the inventory of firm \( i \) in period \( t \). While it seems that the presence
of inventories would intensify the competition leading to lower prices, note that that a
firm, due the quadratic cost assumption, does not have an incentive to substitute future
production with larger current production. Still, note that by the same argument we
have used for the continuous-time case, the collusive price is still an equilibrium price,
since adding inventory is costly and reduces future prices without adding to current
revenues. We leave the formal treatment of this case to a separate work that is in
progress.

7 Concluding remarks

Dynamic games have contributed significantly to our understanding of economic phe-
monena in contexts where static perspectives are believed to conceal critical aspects
of economic behavior. An important literature in the study of firm competition in
oligopolistic settings was initiated by Fershtman and Kamien (1987), showing that,
contrary to the static Cournot prediction, if prices are sticky, firms tend to compete
more intensely and prices to approach lower levels than the Cournot equilibrium. That
sticky price model is based on the assumption that consumer demand evolves dynami-
cally depending on the difference between a notional level of demand and firms’ supply.

In this paper we propose a different approach. We assume that consumer demand
is static but prices are sticky. It follows that, if supply exceeds demand the surplus
will be unsold. This adds an important complication to our model, which cannot be
solved using the traditional methods of smooth analysis. The results of our study are
summarized in a set of conjectures that imply equilibrium prices higher than those
predicted by Fershtman-Kamien and the literature that is based on their approach.
Importantly, the presence of sticky prices tends to reduce the intensity of price compe-
tition, leading to higher (than the Cournot equilibrium) prices. Price stickiness implies
also that the collusive price can be supported as a stable equilibrium outcome without
relying on informal, explicit or tacit, collusion on the part of the rival firms.
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