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Accommodation effects in successive Cournot Oligopolies

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Accommodation effects in successive Cournot Oligopolies

by

Christos Constantatos and Ioannis Pinopoulos

Abstract

When a vertically integrated firm competes against non integrated rivals in the input and the final-good markets, the layman’s belief is that, when the non-integrated downstream firms purchase part of their input from the upstream division of the integrated firm, the downstream division will compete less aggressively, in order to enhance the input sales of its partner. However, most "standard" market interface models of successive Cournot duopolies they ignore this softening of downstream competition.

We show that, when input purchases are allowed to flow continuously, the downstream accommodation effect exists. Even if the upstream firms decide their quantities in advance, the input and final-good markets must clear simultaneously. This, together with Cournot competition in the upstream market imply that, input quantities supplied are observable, but at the moment the independent downstream firm makes its final output decision, the input price cannot be determined yet, otherwise the independent seller is leader in the final-good market. Even, though, the downstream division of the integrated firm cannot by its behavior increase the sales of its upstream partner, it can increase the market-clearing price at which the input will be sold. Considering this effect shows that final-good prices are higher than expected by standard models.

Keywords: Vertically related markets, vertical integration, market interface, accommodation effect, product differentiation

JEL classification: L4, L22

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1 Introduction

Market-interface models of vertical integration, pioneered by Salinger (1988), assume that the input is sold at a uniform price in an open market, where upstream firms participate as sellers and downstream firms participate as buyers. Those models depict markets where there is an asymmetry between up- and downstream firms in the input market: downstream firms behave as competitive buyers (even if they are oligopolists in their final-product market) while the behavior of upstream firms depends on seller’s concentration in the input market. Situations where downstream firms have market power are better described by bargaining-type models. Thus, the typical scenario of market-interface models is as follows: a) upstream firms set input quantities or prices, b) downstream firms purchase inputs, and c) quantity- or price-competition takes place in the output market. Under vertical integration, integrated firms can either participate or completely withdraw from the upstream market. We focus on the case where integrated firms participate in the upstream market as sellers of the input.

Consider the quite common situation where a vertically integrated firm competes against non integrated rivals in the final-good market. The layman’s belief is that, when the non integrated downstream firms purchase, at least part of their input from the upstream division of the integrated firm, the downstream division of the integrated firm will compete less aggressively, in order to support the input sales of its upstream division.

Despite this common belief, the "standard" market interface models fail to admit this softening of downstream competition. In those models, while the upstream part of the integrated firm decides taking into account the impact of its decision on its downstream partner’s profit, the reverse does not happen. In order to explain this asymmetry, consider a successive Cournot

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2 See Church 2004, and 2008 for an extensive review of marker interface models.
4 By "standard" we mean here some of the most influential papers in the field, like Gaudet & Van Long 1996; Higgins 1999; Inderst & Valletti 2007, 2009, to name a few.
5 The softening of downstream competition (or accommodation effect) has been identified in models where upstream firms announce input prices and downstream competition is over prices (Chen 2001; Church, 2008; Hombert et al, 2012). However, the accommodation effect identified in those models is qualitatively different from ours. In those models, the integrated firm has an incentive to price less aggressively in the downstream market in order to affect the input requirements of the unintegrated downstream rival for any given
duopoly, where one firm (firm 1) is vertically integrated. Input quantities supplied are decided at the first stage of the game, while output quantities at the second. When the downstream division of the integrated firm (call it \(D_1\)) decides on its output, the output of its upstream partner (\(U_1\)) has already been decided, therefore there is no point for \(D_1\) to behave as to increase its non integrated rival’s (firm \(D_2\)) demand for the input.

In this paper we vindicate the layman’s belief, by maintaining that the above argument is erroneous, unless input orders are only placed once, before downstream production and sales take place. When input purchases are allowed to flow continuously during the period, even if the upstream firms decide their quantities in advance, the input and final-good markets must clear simultaneously. The latter implies that, at the moment firm \(D_2\) makes its final output decision, either the quantity of input supplied, or the input price must be unknown, otherwise that decision has already been determined. Since the Cournot assumption about the upstream market competition implies that both input sellers be committed in their quantities, and that these quantities are observable, the price of the input must be determined simultaneously with that of the final good.

The implication of the above reasoning is that, while indeed \(D_1\) cannot by its behavior increase the sales of \(U_1\), it can increase the market-clearing price at which the output of \(U_1\) will be sold. For any amount of the final-good \(D_2\) wishes to supply, a reduction of the quantity sold by \(D_1\) will increase the output price, and therefore increase the willingness-to-pay for input of \(D_2\). In other words, by limiting its output, \(D_1\) shifts upwards the derived demand for the input of the non-integrated downstream seller. Since total supply is vertical, this shift will translate to an increase of the input price.

2 The model

Assume a differentiated final good \(Q\) the production of which requires only a homogeneous input \(X\) in one-to-one proportion, all other costs for the

\(^2\) The model

Assume a differentiated final good \(Q\) the production of which requires only a homogeneous input \(X\) in one-to-one proportion, all other costs for the input price. As Church 2008 comments, this effect depends on downstream competition being of Bertrand type. In our model, the integrated firm has an incentive to produce less in the downstream market in order to affect the input price that unintegrated rivals are willing to pay for any given input requirements. It is rather evident, that the latter effect is present in both Cournot and Bertrand competition.
production of $Q$ being equal to zero. Let the marginal production cost of $X$, as well as all costs needed to transform $X$ to $Q$ be constant, and for simplicity assume them equal to zero. We allow for brand-name differentiation of the final good, and assume for simplicity only two variants, 1, 2, sold by two competing downstream firms. The inverse demand for each variant is

$$p_i = p_i(q_i, q_j), \quad i, j = 1, 2, \quad i \neq j,$$  \hspace{1cm} (1)

with $p_i, q_i, q_j \geq 0$, and $\frac{\partial p_i}{\partial q_i} \leq \frac{\partial p_i}{\partial q_j} < 0$, which implies that goods are imperfect substitutes, unless the first relation holds with equality, in which case there are perfect substitutes.

Consider the following two-stage game. There are four players, two upstream firms $U_1, U_2$, producing the input $X$, and two downstream firms $D_1, D_2$, using that input in order to produce two variants of the final good $Q$. At the first stage upstream players simultaneously choose quantities $x_1, x_2 \in \mathbb{R}^+$; at the second stage, having observed $x_i$'s but not $w$, each downstream firm chooses an output schedule $q_i(w) \in \mathbb{R}^+$. Once all $x$'s and $q$'s have been decided, an auctioneer determines the input and final-good prices, $w$, and $p$, respectively, that simultaneously clear the input and final-good markets. Assume that that players $U_1$, and $D_1$, decide to cooperate, maximizing joint profits. It is well known that cooperation among vertically related units implies firstly "transfer pricing", i.e., selling the input internally at marginal cost. Firm $U_1$ may also sell the input in the open market to the rival of its downstream partner, at the prevailing price $w$. Cooperation implies also that when making a decision, each operating unit must also take into account the impact of that decision on its partner unit's profit. Thus, operation-units $U_1$ and $D_1$ choose $x_i, q_i$, respectively, in order to maximize the common objective function of the vertically integrated entity

$$\pi_1 = wx_1 + p_1 q_1$$  \hspace{1cm} (2)

Ideally, all decisions should be made centrally and at the same time, such thing is however impossible due to the timing of the game: downstream decisions must be taken subsequent to the upstream ones.

2.1 Downstream equilibrium

Let $R_2(q_2, q_1) = q_2 p_2(q_2, q_1)$ denote $D_2$'s revenue. The derived demand for the input of $D_2$ is found by equating marginal-revenue product to the
price of the input, \(i.e., MRP (q_2, q_1) \equiv \left( \frac{\partial R_2}{\partial q_2} \right) \left( \frac{\partial q_2}{\partial X_2} \right) = w\), where \(X_2\) represents the amount of input used by \(D2\). Since one unit of input produces one unit of output, the input’s marginal product (second parenthesis above) is equal to 1. Thus, for any level of output \(q_2\) the inverse demand for the input is

\[
w^{D2} (q_2; q_1) = \frac{\partial R_2 (q_2, q_1)}{\partial q_2} \tag{3}\]

with \((\partial^2 R_2 / \partial q_2^2) < 0\), and \((\partial^2 R_2 / \partial q_2 \partial q_1) < 0\), due to product substitutability.

The maximization problem of \(D2\) is

\[
\forall w, \quad \max_{q_2} \pi_{D2} = (p_2 - w) q_2 \tag{4}
\]

which yields the reaction function

\[
q_2 = f_2 (q_1; w) \tag{5}
\]

with both partial derivatives negative. The maximization problem of \(D1\) is

\[
\max_{q_1} \pi_{D1} = p_1 q_1 + x_1 w^{D2} (q_2; q_1). \tag{6}
\]

Note that while \(x_1\) in the above is fixed, thus not representing a choice variable, \(D1\) can affect the price at which \(x_1\) is sold. Setting the derivative equal to zero we get

\[
\partial \pi_{D1} / \partial q_1 = \frac{\partial (p_1 q_1)}{\partial q_1} + x_1 \frac{\partial w^{D2} (q_2; q_1)}{\partial q_1} = 0
\]

Recall that \(\partial w^{D2} (q_2; q_1) / \partial q_1 = \partial R_2^2 / \partial q_2 \partial q_1 < 0\). Let \(q_1 = f_1 (q_2)\) be the solution of the above when \(x_1 = 0\), and write the reaction function as

\[
q_1 \equiv \tilde{f}_1 (q_2; x_1) = f_1 (q_2) + x_1 v (q_2) < f_1 (q_2) \tag{7}
\]

where \(v (q_2) < 0\), due to \((\partial^2 R_2 / \partial q_2 \partial q_1) < 0\). The fact that \(\forall q_2, \tilde{f}_1 (q_2; x_1) < f_1 (q_2)\) implies that the downstream division of an integrated firm behaves less aggressively when it takes into account the profit of its upstream partner. This is expressed by a move of its reaction function to the left. The larger is the amount of \(x_1\) supplied by its upstream partner, the more pronounced this shift is; a standard model ignores this effect.
Solving the system (5), (7), yields the downstream-equilibrium levels of output, as functions of $x_1$ and $w$:

$$
\tilde{q}_1 = \tilde{q}_1 (x_1, w), \quad \tilde{q}_2 = \tilde{q}_2 (x_1, w)
$$

with

$$
\frac{\partial \tilde{q}_1}{\partial x_1} < 0, \quad \frac{\partial \tilde{q}_1}{\partial w} > 0, \quad \frac{\partial \tilde{q}_2}{\partial x_1} > 0, \quad \frac{\partial \tilde{q}_2}{\partial w} < 0.
$$

### 2.2 Upstream equilibrium

Equilibrium in the upstream market requires that total supply $X \equiv x_1 + x_2 = \tilde{q}_2 (x_1, w)$. Solving this for $w$ we obtain the inverse demand function for the input:

$$
w = w (x_1, x_2) = \tilde{w} (X, x_1)
$$

(8)

Since $X$ is total supply, in the above, we have isolated the "supply" effect of $x_1$, from its "demand" effect: the former refers to the impact of $x_1$ on $w$ through increasing total supply, while the latter through increasing demand. Obviously, $\partial \tilde{w} / \partial X < 0$, while $\partial \tilde{w} / \partial x_1 > 0$. Taking (8) into account, we can write $U1$’s maximization program as

$$
\max_{x_1} \pi_{U1} = \tilde{w} (X, x_1) \cdot x_1 + \tilde{p}_1 (x_1, \tilde{w} (X, x_1)) \cdot \tilde{q}_1 (x_1, \tilde{w} (X, x_1))
$$

(9)

Deriving the RHS of the above obtains:

$$
\frac{\partial \pi_{U1} (x_1, x_2)}{\partial x_1} = \tilde{w} (X, x_1) + x_1 \left( \frac{\partial \tilde{w}}{\partial X} + \frac{\partial \tilde{w}}{\partial x_1} \right) + \tilde{q}_1 \cdot \left[ \frac{\partial \tilde{p}_1}{\partial x_1} + \frac{\partial \tilde{p}_1}{\partial w} \left( \frac{\partial \tilde{w}}{\partial X} + \frac{\partial \tilde{w}}{\partial x_1} \right) \right]
$$

$$
+ \tilde{p}_1 \cdot \left[ \frac{\partial \tilde{q}_1}{\partial x_1} + \frac{\partial \tilde{q}_1}{\partial w} \left( \frac{\partial \tilde{w}}{\partial X} + \frac{\partial \tilde{w}}{\partial x_1} \right) \right]
$$

(10)

Ignoring the downstream accommodation effect eliminates in (10) all the
direct effects of $x_1$ as well as all its effects through $\tilde{w}$, leaving only the non-signed terms:

$$
\frac{\partial \pi_{U1S}(x_1, x_2)}{\partial x_1} = \tilde{w}(X, x_1) + x_1 \frac{\partial \tilde{w}}{\partial X} + \left( q_1 \frac{\partial \bar{p}_1}{\partial \tilde{w}} + \bar{p}_1 \frac{\partial q_1}{\partial \tilde{w}} \right) \frac{\partial \tilde{w}}{\partial X} + \]

(11)

where $\pi_{U1S}(x_1, x_2)$ represents the objective of the upstream firm in a game where downstream accommodation is ignored. The first two terms correspond to marginal revenue, while the third represents a strategic upstream-accommodation effect: increases in $\tilde{w}$ result in both, higher price and higher quantity for $D1$, therefore, in the classic model the upstream division of the integrated firm behaves less aggressively in the open market for input, compared to its non-integrated rival. Compared to (11), the RHS of (10) identifies five new additional strategic effects in the decision of $U1$, all but one of them, positive. Unfortunately, we cannot identify the relative magnitude of all the effects in (10) without imposing more structure in the model; this task is undertaken in the next section. Meanwhile, the list below identifies all the new strategic effects in the decision of $U1$:

- $x_1 \frac{\partial \tilde{w}}{\partial x_1}$: Impact of coordination on input price. $U1$ recognizes that higher amounts of $x_1$ induce downstream accommodation thus raising the input price.

- via $\tilde{q}_1$:
  - $\tilde{q}_1 \frac{\partial \bar{p}_1}{\partial x_1}$: Direct impact of coordination on downstream price. $U1$ recognizes that a slightly higher amount of $x_1$ increases the price of $q_1$ by reducing its quantity.

  - Impact of coordination on downstream price via a change in the price of the intermediate good: a slightly higher amount of $x_1$ increases, ceteris paribus, the price of the input, and through this, also $p_1$.

- via $\tilde{p}_1$:
  - $\tilde{p}_1 \frac{\partial q_1}{\partial x_1}$: Direct effect of coordination on downstream quantity.

  - Indirect effect of coordination on downstream quantity through affecting the input price: by increasing $\tilde{w}$ (through the reduction of $\tilde{q}_1$), a slight increase in $x_1$ increases $\tilde{q}_1$.
Finally, note that the first order condition of the independent supplier $U_2$ is also affected by the downstream accommodation effect. While there are no strategic effects in the behavior of $U_2$, the strategic effects in the behavior of $U_1$ affect the residual demand $U_2$ faces, and therefore its reaction function. The latter represents the best reply to any quantity hypothetically chosen by $U_1$, and is derived using the resulting residual demand. However, as $x_1$ increases, market demand increases as well, hence the reduction of the residual demand is partially offset.\(^6\)

3 The Bowley utility function case

In order to further investigate the behavior of the participants in both markets, in this section we maintain the setting of the previous section, and in addition we further specify downstream demand, by assuming it takes the form that derives from the well-known Bowley utility function; i.e., we replace (1) by the following

$$p_i = 1 - q_i - \theta q_j, \quad i, j = 1, 2, \quad i \neq j,$$

with $p_i, q_i, q_j \geq 0$, and $\theta \in [0, 1].\(^7\)$

3.1 Downstream equilibrium

Firm 2’s revenue now becomes $R_2 = q_2 (1 - q_2 - \theta q_1)$ yielding the inverse demand function for input

$$w(q_1, q_2) = 1 - 2q_2 - \theta q_1.$$

\(^6\)The outward shift in $U_2$’s reaction function is obviously not parallel, but pivoting upwardly around the point that corresponds to $x_1 = 0$, since higher levels of $x_1$ imply also more important shifts of the total market demand. Theoretically, the reaction function of $U_2$ may not only pivot upwardly, but also become upward sloping. We do not investigate the issue, since in the standard case presented below, such thing never happens.

\(^7\)The above demand function derives from the well-known Bowley utility function. When $\theta = 1$ products are homogeneous, while $\theta = 0$ implies that the two versions of $Q$ do not compete in the same market.
On the other hand, maximizing firm 2’s profit function with respect to \( q_2 \) yields its output reaction function,

\[
q_2 = \frac{1}{2} \left( 1 - w - \theta q_1 \right)
\]  

(13)

The integrated firm’s downstream division maximizes (2) taking \( x_1 \) as given, which yields the reaction function (corresponding to (7)):

\[
q_1(q_2; x_1) = \frac{1 - \theta x_1}{2} - \frac{\theta}{2} q_2
\]  

(14)

Obviously, in terms of (7), the part \( \nu(x_1) = -\left(\frac{\theta}{2}\right) x_1 \), which shifts the reaction function parallel to the left, and the shift is proportional to \( x_1 \). Solving (13), (14) simultaneously, yields the equilibrium quantities in the downstream market

\[
q_1 = \frac{(2 - \theta) + w(2 - x_1)}{4 - \theta^2}, \quad q_2 = \frac{(2 - \theta) - 2w + x_1 \theta^2}{4 - \theta^2}
\]  

(15)

Observe in the above that despite the presence of \( w \), which ultimately depends on \( x_1 + x_2 \), equilibrium quantities also depend on \( x_1 \), and recall that, at this moment, \( x_1 \) is observable, but \( w \) is not: it will be determined as the outcome of the game. Thus, what each downstream firm has chosen up to now, is a correspondence between announcements of \( x_1 \) and functions \( q_i(w) \).

Note that

\[
\frac{\partial q_1}{\partial w}(\theta) = \frac{\theta}{4 - \theta^2} > 0, \quad \frac{\partial q_1}{\partial x_1}(\theta) = -\frac{2\theta}{4 - \theta^2} < 0
\]

\[
\frac{\partial q_2}{\partial w}(\theta) = \frac{-2}{4 - \theta^2} < 0, \quad \frac{\partial q_2}{\partial x_1}(\theta) = \frac{\theta^2}{4 - \theta^2} > 0
\]

The impact of \( w \) on \( q_1, q_2 \), is as expected: positive and negative, respectively, since an increase in \( w \) results in an asymmetric increase of \( D2 \)’s marginal cost. In absolute terms both derivatives are increasing in \( \theta \). Note that \( \frac{\partial q_1}{\partial w} < \frac{\partial q_2}{\partial w} \), \( \frac{\partial q_1}{\partial w}(0) = 0, \frac{\partial q_2}{\partial w}(0) = -1/2 \), since the impact of \( w \) on \( q_1 \) is only due to strategic effects that vanish when the two products become completely independent, while the corresponding effect on \( q_2 \) contains also a direct effect on cost. The novelty of our approach focuses on the other two derivatives, depicting the independent impact of \( x_1 \) (as opposed to its impact through \( w \)). Clearly, \( x_1 \) creates an accommodating effect in the downstream market,
not previously noted in the literature: if \( U_1 \) announces higher amounts of \( x_1 \), the equilibrium output of \( D_1 \) is reduced, yielding market share to \( D_2 \). This effect is more pronounced, the lower the degree of differentiation (higher \( \theta \)), and vanishes when the two products are independent (\( \theta = 0 \)).

The profit of the operating unit \( D_1 \) and that of \( D_2 \) are given below as functions of \( w, x_1 \), are:

\[
\pi_{D_1} = p_1q_1 = \frac{(w\theta - 2x_1\theta - \theta + 2) \left[ x_1\theta \left(2 - \theta^2\right) + (w - 1)\theta + 2\right]}{(4 - \theta^2)^2},
\]

\[
\pi_{D_2} = \frac{(x_1\theta^2 - \theta - 2w + 2)^2}{(\theta^2 - 4)^2} \tag{16}
\]

### 3.2 Upstream equilibrium

Since \( D_2 \) is the sole purchaser of \( X \) in the open market, equilibrium in that market requires that \( x_1 + x_2 = q_2 \). Substituting \( q_2 \) from (15) and solving for \( w \), we obtain the inverse derived demand for input in the open market:

\[
w = \frac{2 - \theta}{2} - \frac{(2 - \theta)(\theta + 2)}{2} x_2 - (2 - \theta^2) x_1 \tag{17}
\]

The impact of \( x_1, x_2 \), on \( w \) is not symmetric: letting \( \nu_2 \equiv \frac{(2-\theta)(\theta+2)}{2} \), \( \nu_1 \equiv (2 - \theta^2) \), we see that \( \nu_2 - \nu_1 = (\theta^2/2) > 0 \), the difference depending on the degree of differentiation. This is expected since increases in \( x_1 \) are followed by a more accommodating behavior by \( D_1 \).

Substituting \( w \) by (17) and \( p_1q_1 \) by (16) into (2), we obtain the objective function of \( U_1 \):

\[
\pi_{U_1} = (\theta^2 - 2) x_1^2 + (-\theta + x_2 (\theta^2 - 2) + 1) x_1 + \frac{1}{4}(x_2\theta - 1)^2 \tag{18}
\]

Maximizing the above yields \( U_1 \)'s reaction function:

\[
x_1 = \frac{1}{2} \left( \frac{1 - \theta}{2 - \theta^2} - x_2 \right) \tag{19}
\]

Using (17) firm \( U_2 \)'s profit can be written as:

\[
\pi_{U_2} = -\frac{1}{2}(2 - \theta)(2 + \theta)x_2^2 + \left(\frac{2 - \theta}{2} - x_1 (2 - \theta^2)\right) x_2 \tag{20}
\]
from which we derive the reaction function

\[ x_2 = \frac{2 - \theta - 2x_1(2 - \theta^2)}{2(4 - \theta^2)} \]  

(21)

The upstream equilibrium is found by solving the system (19), and (21). In the appendix we show that for this system to yield nonnegative quantities it must be that \( \theta < \tilde{\theta} < \sqrt{3} - 1 \approx 0.73205 < 1 \), implying that when the final good is homogeneous, or brand differentiation is not sufficiently important, \( U1 \) wishes to withdraw from selling the input in the open market. Assuming \( \theta < \tilde{\theta} \), we derive the equilibrium upstream quantities and the resulting input price: \(^8\)

\[ x_1(\theta) = \frac{\theta^3 - 6\theta + 4}{2\theta^4 - 16\theta^2 + 24}, \quad x_2(\theta) = \frac{1}{6 - \theta^2} \]  

(22)

and

\[ w(\theta) = \frac{1}{2} - \frac{1}{6 - \theta^2} \]  

(23)

Equipped with the equilibrium value of \( x_1 \) and \( w \), we can derive the entire-game equilibrium values of the downstream market, namely quantities

\[ q_1(\theta) = \frac{2 - \theta}{2(2 - \theta^2)}, \quad q_2(\theta) = \frac{8 - \theta[(2 - \theta)\theta + 6]}{2(\theta^4 - 8\theta^2 + 12)}, \]  

(24)

and prices

\[ p_1(\theta) = \frac{1}{2} \left(1 - \frac{\theta}{6 - \theta^2}\right), \quad p_2(\theta) = \frac{1}{2} \left(1 + \frac{1 - \theta}{2 - \theta^2} - \frac{1}{6 - \theta^2}\right). \]  

(25)

From (24) we get that

\[ \frac{q_1}{q_2} = 1 + \frac{4}{8 - \theta[(2 - \theta)\theta + 6]} \]

where the denominator of the fraction is positive, \( \forall \theta \in [0, 1] \), hence, \( q_1(\theta) > q_2(\theta) \): despite the accommodation effect, \( D1 \) obtains a larger market share due to its lower cost. \(^9\) Similarly, from (25) it is easy to show that \( p_1(\theta) - p_2(\theta) = 2(1 - \theta)/(12 - 8\theta^2 + \theta^4) > 0 \).

\(^8\)It can be verified that the numerator of \( x_1 \) in the expression below is positive \( \forall \theta \in [0, \tilde{\theta}] \), as is the denominator \( \forall \theta \in [0, 1] \).

\(^9\)Recall that \( D1 \) receives the input internally at marginal cost, which, in this case, is zero.
4 Comparisons

We further investigate the implications of the accommodation model by comparing its features to those of the standard model where the accommodation effect is ignored. The main difference between the standard model and the one presented in the previous section is that instead of (6), the objective of the downstream division of the integrated firms is assumed to be:

$$\max_{q_1} \pi_{D1} = p_1 q_1 + x_1 w$$

where both $x_1$ and $w$ are assumed to be already determined. As said earlier, our main objection to this approach is that in order for both, the upstream quantities and the input price to be determined, the input quantity used by $D2$ is also determined, and so is its output. This reasoning implies that $q_2$ has been determined before $q_1$, i.e., in the downstream game $D2$ is a Stackelberg leader! Nevertheless, in this section we put our objection aside and solve the standard model in order to compare its outcome to the accommodation model. Since all the steps to the solution follow the same path, presented in the previous section, in this section we simply contrast the results in the following table. Hereafter, all functions and equilibrium values referring to the "standard" and "accommodation" models will be indicated by superscripts "S" and "A, respectively.
shown that
$x^S(x_1, x_2) = \frac{1}{3} (\theta - 2)((x_1 + x_2) - x_1)^2$}

\[D1's \ reaction \ fun.\]
\[q_1^S (q_2) = \frac{1}{2} - \frac{\theta}{2} q_2\]
\[2nd-stage \ equil. \ q_1\]
\[q_1^S (w) = \frac{(2 - \theta - 2 \theta)}{4 \theta^2}\]
\[2nd-stage \ equil. \ q_2\]
\[q_2^S = \frac{2 - \theta}{4 \theta^2 - 2 \theta + 1}\]

\[Inv. \ derived \ dem.\]
\[w^S (x_1, x_2) = \frac{1}{2} (\theta - 2)((x_1 + x_2) - x_1)^2\]

\[U1's \ reaction \ fun.\]
\[x_1^S = \frac{2(1 - \theta - 2 \theta)(2 \theta - 2 \theta)}{8 - 3 \theta^2}\]

\[U2's \ reaction \ fun.\]
\[x_2^S = \frac{1}{2} \left( \frac{1}{8} - x_1 \right)\]

\[Equil. \ value \ of \ x_1\]
\[x_1^S (\theta) = \frac{\theta (2 + 2 \theta) - 2}{2 \theta (2 + 2 \theta)}\]

\[Withdrawal \ at \ \theta\]
\[\theta^S \approx 0.732051\]

\[Equil. \ value \ of \ x_2\]
\[x_2^S (\theta) = \frac{1}{2} \left( \frac{1}{8 \theta^2} \right) + \frac{1}{8 \theta^2}\]

\[Equil. \ value \ of \ w\]
\[w^S (\theta) = \frac{8 - \theta (4 - \theta)}{8(3 - \theta^2)}\]

\[Equil. \ value \ of \ q_1\]
\[q_1^S (\theta) = \frac{(2 - \theta)(4 - \theta)}{8(3 - \theta^2)}\]

\[Equil. \ value \ of \ q_2\]
\[q_2^S (\theta) = \frac{4 - \theta}{8(3 - \theta^2)}\]

\[Equil. \ value \ of \ p_1\]
\[p_1^S (\theta) = \frac{(2 - \theta)(4 - \theta)}{8(3 - \theta^2)}\]

\[Equil. \ value \ of \ p_2\]
\[p_2^S (\theta) = \frac{16 - \theta (4 - \theta) (\theta + 6)}{8(3 - \theta^2)}\]

\[Equil. \ profits \ of \ U1\]
\[\pi_U^S = \frac{(2 - \theta)(4 - \theta) - [2 - \theta (\theta + 2)]}{16(\theta + 2)(3 - \theta^2)}\]

\[Equil. \ profits \ of \ D1\]
\[\pi_{D1}^S = \frac{(2 - \theta) (\theta + 6)^2 - [2 - \theta (\theta + 2)]}{64(3 - \theta^2)}\]

\[Total \ int. \ profits\]
\[\pi_1^S = \frac{(2 - \theta)(4 - \theta) (\theta + 6) + [2 - \theta (\theta + 2)]}{16(\theta + 2)(3 - \theta^2)}\]

\[Equil. \ profits \ of \ U2\]
\[\pi_U^S = \frac{(2 - \theta)(\theta - 2) (\theta + 6)^2}{32(3 - \theta^2)}\]

\[Equil. \ profits \ of \ D2\]
\[\pi_{D2}^S = \frac{(4 - \theta) (\theta + 6)^2}{16(3 - \theta^2)}\]

\[Standard \ model\]
\[\pi_U^A = \frac{(2 - \theta)(4 - \theta) - [2 - \theta (\theta + 2)]}{16(\theta + 2)(3 - \theta^2)}\]

\[Accommodation \ model\]
\[\pi_U^A = \frac{(2 - \theta)(4 - \theta) - [2 - \theta (\theta + 2)]}{16(\theta + 2)(3 - \theta^2)}\]

Let us start from $U1$’s reaction function in the two models. It can be shown that $x_1^U (x_2) / x_1^S (x_2) = \frac{3}{4} + \frac{1}{4 \theta^2}$, which is increasing in $\theta$, and equal to 1 for $\theta = 0$. Hence, $\forall \theta \in [0, 1]$, and $\forall x_2 \geq 0$, downstream accommodation makes $U1$ more aggressive in the input market. On the other hand $x_2^A (x_1) - x_2^S (x_1) = (\theta^2 x_1) / (8 - 2 \theta^2) > 0$, which implies that firm $U2$ also becomes more aggressive in the A model. We can immediately conclude that total
the total amount of input put for sale in the A model is larger than the corresponding amount in the S model.

Looking at the equilibrium input price, we have
\[ w^S - w^A = \frac{\theta^2 (\theta^3 - 6\theta + 4)}{8(\theta^4 - 9\theta^2 + 18)} > 0, \forall \theta < \sqrt{3} - 1 = \bar{\theta}; \] despite the higher input quantities supplied by both firms, the input price is higher in model A, due to higher downstream demand. Firm $D2$ increases its output so much as to cause a rise in the input price, relative to the S model. However, the reduction of $q_1$ is so substantial, that the price of the final good can be shown to increase for both varieties. Hence, any benefits to consumers due to vertical integration are reduced due to the fact that full coordination of the two stages softens downstream competition.

Concerning profits, both the upstream and the downstream rivals of the integrated firm benefit from coordination. The difference in the profit of the integrated firm between the two models requires a rather tedious analysis. Instead, the following diagram presents a numerical analysis of that difference, based on all the admissible values of $\theta$.

Obviously, playing softly in the downstream market where competition is in strategic substitutes yields profits to the rival. When products are strongly differentiated, this effect is not so important, and is outweighed by the superior performance due to better coordination of the two stages. As product differentiation is reduced, though, the integrated firm wishes to send to its downstream rival the message of abandoning strict coordination between its up and downstream divisions, but, unless we introduce some commitment possibility in the model, this message is not credible.
5 Conclusions

We have considered a successive Cournot oligopoly with one integrated firm facing a separate rival in each market. Our main contribution relative to the existing literature is to recognize that the downstream division of the integrated firm may wish to accommodate its rival in order to increase the profit of its upstream partner. Previous literature has ignored this effect arguing that, at the moment downstream decisions are taken, the upstream market is already cleared. Such an assumption, however, would lead to a von Stackelberg situation in the downstream market with the independent firm assuming the leader’s role. While such a scenario is plausible when input purchases must be ordered way in advance, it is problematic when the input can be continuously delivered to the downstream firms. Hence, when the downstream market is no capacity constrained, the downstream division must consider the strategic impact of its decision on the profits of its upstream partner, exactly the same way as does the upstream division towards it.

Taking this effect into account changes the outcome of the game. In this paper we simply performed a comparative statics analysis between the model as it has been formulated in the literature, and the version that recognizes full coordination between the two divisions of the integrated firm. Currently we are investigating the validity of some results already presented in the literature, when our full-coordination model is considered.

6 References


Church, J., Vertical Mergers, in "Issues in competition law and policy" 1455 (ABA Section of Antitrust Law 2008) Chapter 61.


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