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Upstream horizontal mergers and vertical integration

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Upstream horizontal mergers and vertical integration* 

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Abstract

We study upstream horizontal mergers when one of the merging parties is a vertically integrated firm. Under upstream cost symmetry and observable contracting, we demonstrate that such type of horizontal mergers always harm consumers through a vertical partial foreclosure effect. Under observable contracting but upstream asymmetric costs, we show that overall consumer surplus may increase due to the merger even though input prices increase and some consumers are worse off. Under upstream cost symmetry but unobservable contracting, we find that consumers may be better off as a result of the merger even in the absence of exogenous cost-synergies between the merging firms. In all cases under consideration, the merger is always profitable for the merging parties.

Keywords: Vertical relations; vertical integration; horizontal mergers; consumer surplus

JEL Classification Codes: L11; L13; L41; L42

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1. Introduction

A classic topic of antitrust economics is the welfare effects of horizontal mergers – that is mergers between competitors. Since vertical relations are ubiquitous in real-world markets, it is nowadays widely acknowledged, by both economic theorists and antitrust agencies, that the vast majority of horizontal mergers take place in either the upstream or the downstream sector of vertically related industries.

In this paper we study upstream horizontal mergers. A key aspect of our analysis is that one of the merging parties is a vertically integrated firm. In other words, one insider party to the upstream merger is also present in the downstream market through a subsidiary. This assumption is primarily motivated by one of the largest oil mergers ever, namely the BP/ARCO merger. In 1999, British Petroleum Amoco (BP) announced its intention to acquire the Atlantic Richfield Company (ARCO). While BP and ARCO were present in the Alaskan North Slope (ANS) – the upstream market for crude oil -, ARCO was also present downstream in West Coast refining and marketing. Moreover, ARCO used all its own ANS production for its own refineries and BP was a major supplier to ARCO’s competitors, such as Chevron and Tosco.

As Bulow & Shapiro (2002) comment, the basic downstream antitrust concern in the BP/ARCO merger “was whether the acquisition of ARCO would allow BP to elevate the price of ANS crude oil to West Coast refineries. Ultimately, higher ANS crude oil prices might lead to higher prices of refined products, especially gasoline, on the West Coast.” The objective of this paper is to provide a formal explanation of the aforementioned antitrust issue.

We consider a two-tier market consisting of two competing vertical chains. In each chain, there is a single upstream firm that produces an input which a single downstream firm uses in one-to-one proportion in the production of a differentiated final good. We assume that one vertical chain is vertically integrated whereas the other chain is vertically separated. At some point, the vertically integrated chain (or vertically integrated firm) considers merging with the...
upstream independent input supplier. Such a merger is classified as horizontal, since both merging entities are present in the upstream market, it has nevertheless important vertical implications since the independent downstream firm must now purchase its input from the upstream counterpart of its rival in the downstream market.

The timing of moves is as follows. At the first stage, the vertically integrated firm and the independent upstream supplier decide whether to merge or not. At the second stage, the independent upstream supplier (if the merger does not occur) or the newly merged firm (if the merger occurs) makes the independent downstream firm a take-it-or-leave-it, two-part tariff contract offer; the contract consists of an input price and a fixed fee. In the pre-merger case, the contract stipulated in the vertically separated chain can be either observable (observable contracting) or unobservable (unobservable contracting) by the integrated firm. By construction of the model, there is no issue regarding contract observability in the post-merger case. At the last stage, downstream competition takes place a la Cournot.

In the baseline model, we assume upstream cost symmetry and observable contracting. Under a general demand function, we show that the upstream horizontal merger creates partial vertical foreclosure: the input price paid by the independent downstream firm will increase, thereby yielding greater market share to the downstream affiliate of the horizontally merged entity.\(^3\) This translates the higher input price into higher final-good prices and lower total output, making consumers worse off.

Our contribution is that we formally incorporate the aforementioned foreclosure effect, which is well-established in the literature on vertical mergers, into horizontal merger analysis.\(^4\) Under our modelling structure, this effect is, in fact, the only effect at play: since in the pre-merger case all of the vertically integrated firm’s upstream production is directed to its downstream affiliate (captive sales) the merger does not affect concentration in the upstream merchant market, which is monopolized in both the pre- and the post-merger situation.

We consider two modifications of the baseline model under which consumer surplus may increase due to the merger. First, we maintain the assumption of observable contracting, however, we introduce upstream cost asymmetry. We assume that, in the post-merger situation, the more efficient firm transfers its technology to the less efficient firm so that the

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\(^3\) Foreclosure is partial in the sense that the independent downstream firm pays a higher input price and produces less of the final good in the post-merger case, however, it is not driven out of the market (in which case foreclosure is complete or full).

\(^4\) For overviews on the potential foreclosure effects of vertical mergers, see Riordan (2005) and Church (2008).
merger generates efficiency gains.\textsuperscript{5,6} In such setting, we show that overall consumer surplus may increase due to the merger even though the input price always increases and some consumers are worse off. Second, we maintain the assumption of upstream cost symmetry, however, we assume unobservable contracting. In that case, we find that the input price may decrease and consumer surplus may increase as a result of the merger even in the absence of exogenous cost-synergies between the merging firms. In both aforementioned cases, we restrict attention to a linear demand function.

Under \textit{upstream cost asymmetry}, the effect of the merger on input price, as well as on final-good prices, crucially depends on which firm - the independent upstream firm or the upstream division of the vertically integrated firm – is more cost-efficient in the pre-merger situation. When the independent upstream firm is \textit{less} efficient than the upstream division of the vertically integrated firm, the merger creates efficiency gains in the upstream production that is directed to the independent downstream rival causing the input price to fall. This effect works against the aforementioned vertical foreclosure effect and when it is sufficiently large it may outweigh the latter, resulting in lower input and final-good prices, thus benefiting all consumers.

When the independent upstream firm is \textit{more} efficient than the upstream division of the vertically integrated firm, on the one hand, the merger does not lower the cost of the upstream production directed to the independent downstream firm, leaving at play only the vertical foreclosure effect. Hence, as in the case of upstream cost symmetry, the input price always increases pulling with it the final-goods prices. On the other hand, the merger creates efficiency gains in the upstream production directed to the downstream division of the merged firm, thus tending to decrease both final-good prices. As it turns out, the final-good price of the independent downstream firm always increase due to the merger irrespective of the magnitude of the efficiency gains. However, when the efficiency gains are sufficiently large, the final-good price of the vertically integrated firm may decrease and consumer surplus may increase as a result of the merger. In other words, overall consumer surplus may increase due to the merger even though the input price always increases and some consumers are worse off. Second, we maintain the assumption of upstream cost symmetry, however, we assume unobservable contracting. In that case, we find that the input price may decrease and consumer surplus may increase as a result of the merger even in the absence of exogenous cost-synergies between the merging firms. In both aforementioned cases, we restrict attention to a linear demand function.

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\textsuperscript{5}A central objective of antitrust authorities all over the world is to consider whether or not efficiency gains associated with horizontal mergers are likely to offset the enhanced market power of the merging firms (see footnote 2). See Williamson (1968) for a classic analysis of the tradeoff between market power and efficiency gains, as well as Röller et al. (2001) for a review of the literature on the efficiency gains of horizontal mergers in one-tier industries.

\textsuperscript{6}It is well-known that upstream cost differences are important and prevalent in natural resource industries. For instance, in the oil and gas industry, the costs of extracting crude oil differ significantly between producers (Gaudet et al., 1999). Assuming that the post-merger entity inherits the lowest cost of the merging parties is problematic when the pre-merger marginal cost differences are due to site-related extraction costs. Nevertheless, to the extent that marginal cost differences are also due to factors other than site-specific ones - for instance due to differences in the extracting method - this assumption works as fair simplification of the situation.
increase due to the efficiency gains entailed by the merger even though some consumers – those buying from the independent downstream firm - are worse off.

Under unobservable contracting in the pre-merger situation, the two-part tariff contract stipulated in the separated chain loses its pre-commitment value thereby eliminating any strategic effect on the integrated firm’s behavior and resulting in upstream marginal-cost pricing. The upstream merger restores the commitment value of the contract. In particular, the post-merger input price is chosen to maximize overall industry profits. When the downstream division of the integrated firm is sufficiently less cost-efficient than the independent downstream firm, it is optimal for the merged firm to set an input price below upstream marginal cost thereby shifting final-good sales to the more profitable downstream rival. The reduction in input price ultimately leads to a reduction in final-good prices; the upstream merger increases consumer surplus even though it does not increase efficiency in the merging firms.

In all cases under consideration, the merger is always profitable. In the pre-merger case, the input price is chosen so as to maximize the vertically separated chain’s profits rather than total industry profits. In the post-merger case, however, the input price is chosen so as to maximize overall industry profits. Therefore, it must hold that overall industry profits increase as a result of the merger. Since overall industry profits increase, and the independent downstream firm’s net profits remain unaffected (in both cases are equal to zero), it must hold that the combined net profits of the vertically integrated firm and the independent upstream supplier increase, implying that the merger is beneficial for the merging parties.

There is a growing literature on the effects of upstream mergers in vertically separated markets. This literature, which begins with the seminal work of Horn & Wolinsky (1988), also includes, among others, Ziss (1995), Fumagalli & Motta (2001), Inderst & Wey (2003), O’Brien & Shaffer (2005), Milliou & Petrakis (2007) and Milliou & Pavlou (2013). Two key insights from these studies are that (i) upstream mergers are profitable and beneficial to consumers only when they entail efficiency gains, and (ii) once efficiency gains are taken into account, a reduction in input price is always a necessary condition for an increase in consumer surplus. Our analysis reveals that when one of the merging parties is a vertically integrated firm - a case which, to the best of our knowledge, has not been formally examined

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by the existing literature -, (i) upstream mergers can be profitable and beneficial to consumers
even in the absence of any efficiency gains, and (ii) once efficiency gains are taken into
account, a reduction in input price is not always a necessary condition for an increase in
consumer surplus.

The rest of the paper is organized as follows. In Section 2, we describe the baseline model
under upstream cost symmetry and observable contracting. In Section 3, we perform the
equilibrium analysis and derive our main results. In Section 4, we modify the baseline model
by introducing upstream cost asymmetry, whereas in Section 5, we modify the baseline
model by considering the case of unobservable contracting. Section 6 concludes the paper.

2. The baseline model with upstream cost symmetry and observable contracting

We consider a vertically related market initially consisting of two competing vertical
chains. In each chain, i = 1, 2, there is a single upstream firm, Ui, that produces an input
which a single downstream firm, Di, uses in one-to-one proportion in the production of a
differentiated final good. We assume that chain 1 is vertically integrated, whereas chain 2 is
vertically separated, i.e., there is the vertically integrated firm U1-D1, one independent
upstream supplier U2 and one independent downstream firm D2.

Marginal production costs in the upstream market are denoted by cUi. We assume that
cu1 = cu2 = cu, so the upstream division of the integrated firm and the independent upstream
supplier are equally efficient as input providers. Marginal transformation costs in the
downstream market are denoted by cDi. No further assumptions are made regarding the
relationship between cD1 and cD2.

We then consider the case where the independent upstream supplier U2 and the vertically
integrated firm U1-D1 contemplate merging to form a new entity, denoted as firm I. Such
merger is qualified as horizontal since both firms are present in the upstream market, it has,
nevertheless, an important vertical aspect in that U2 is the input supplier of U1-D1’s rival in
the downstream market. The assumption of upstream cost symmetry implies that the merger
does not generate efficiency gains, thus allowing to focus on the implications of the vertical
relationship.8

8In Section 4, we explore the role of efficiency gains by considering the case of upstream asymmetric costs.
Suppose that $U(q_1, q_2)$ is a differentially strictly concave utility function and let $q = (q_1, q_2)$. The representative consumer maximizes $U(q) - pq$ giving rise to an inverse demand system $p_i = p(q_i, q_j), \ i, j = 1, 2, i \neq j$, which is twice continuously differentiable. Inverse demands will be downward sloping, $\partial p_i / \partial q_i < 0$, and symmetric cross effects will be negative, $\partial p_i / \partial q_j = \partial p_j / \partial q_i < 0$, implying that final-goods are substitutes. We also assume that the own effect is larger than the cross effect, that is $|\partial p_i / \partial q_i| > |\partial p_j / \partial q_j|$.

We model market interactions as a three-stage game with timing as follows. At the first stage, firms $U1-D1$ and $U2$ decide whether to merge or not. At the second stage, the independent supplier $U2$ (if the merger does not occur) or firm $I$ (if the merger occurs) makes $D2$ a take-it-or-leave-it, two-part tariff contract offer; the contract consists of an input price $w$ and a fixed fee $F$. If there is no merger, we assume that the contract stipulated in the vertically separated chain is observable by the integrated firm. At the last stage, downstream competition takes place a la Cournot. For notational reasons, we use superscripts $S$ or $M$ to denote, respectively, the pre- and the post-merger case.

3. Equilibrium outcomes in the baseline model

3.1. The pre-merger case

Working backwards, we start by solving the last stage of the game. Firms $U1-D1$ and $D2$ choose simultaneously and independently their final-good outputs to maximize profits:

$$\max_{q_1} \pi_{U1-D1} = p_1(q_1, q_2)q_1 - (c_{D1} + c_U)q_1,$$
$$\max_{q_2} \pi_{D2} = p_2(q_1, q_2)q_2 - (w + c_{D2})q_2 - F.$$

The first order conditions of the above maximization problems are given by,

$$p_1 + q_1 \frac{\partial p_1}{\partial q_1} = c_{D1} + c_U,$$  \hspace{1cm} (1)

and

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8In Section 5, we consider the case where, in the pre-merger situation, the contract stipulated in the vertically separated chain is unobservable by the vertically integrated firm.
\[ p_2 + q_2 \frac{\partial p_2}{\partial q_2} = c_{D2} + w, \]  

(2)

respectively. We make the following three assumptions:

Assumption 1. \( \frac{\partial^2 \pi_{U1-D1}}{\partial q_1^2} < 0 \) and \( \frac{\partial^2 \pi_{D2}}{\partial q_2^2} < 0 \).

Assumption 2. \( \frac{\partial^2 \pi_{U1-D1}}{\partial q_1 \partial q_2} < 0 \) and \( \frac{\partial^2 \pi_{D2}}{\partial q_2 \partial q_1} < 0 \).

Assumption 3. \( \frac{\partial^2 \pi_{U1-D1}}{\partial q_1^2} + \left| \frac{\partial^2 \pi_{U1-D1}}{\partial q_1 \partial q_2} \right| < 0 \) and \( \frac{\partial^2 \pi_{D2}}{\partial q_2^2} + \left| \frac{\partial^2 \pi_{D2}}{\partial q_2 \partial q_1} \right| < 0 \).

Assumption 1 guarantees that the second order conditions of the above maximization problems are satisfied. Assumption 2 implies strategic substitutability: firms’ best-response functions in the downstream market are downward sloping, i.e., \( dq_i/dq_j < 0 \). Assumption 3 implies that the best-response functions are well-behaved and have slope less than one, \( |dq_i/dq_j| < 1 \), and therefore there exist unique and stable Cournot equilibria.

Solving together (1) and (2), we obtain the last-stage subgame equilibrium final-good outputs and prices as functions of the input price: \( \hat{q}_1(w), \hat{q}_2(w), \hat{p}_1(w) = p_1[\hat{q}_1(w), \hat{q}_2(w)] \) and \( \hat{p}_2(w) = p_1[\hat{q}_1(w), \hat{q}_2(w)] \). As shown in Appendix A, these last-stage subgame equilibrium outcomes have the following properties:

\[ \frac{d\hat{q}_1}{dw} > 0, \quad \frac{d\hat{q}_2}{dw} < 0, \quad \frac{d\hat{Q}}{dw} < 0, \quad \frac{d\hat{p}_1}{dw} > 0, \quad \frac{d\hat{p}_2}{dw} > 0. \]  

(3)

Next, we solve the second stage of the game in order to determine the equilibrium contract terms. The independent upstream firm \( U2 \) uses the fixed fee to fully extract \( D2 \)’s profits,

\[ F = (\hat{p}_2(w) - w - c_{D2})\hat{q}_2(w), \]  

(4)

and thus sets the input price so as to maximize,
\[
\max_w \pi_{U_2} = (w - c_u) \hat{q}_2(w) + F = (\hat{p}_2(w) - c_u - c_{D_2}) \hat{q}_2(w).
\] (5)

It can be seen from (5) that the input price is chosen so as to maximize the unintegrated vertical chain’s profits. The first order condition of the above maximization problem, after using (2), is given by:

\[
(w - c_u) \frac{d\hat{q}_2}{dw} + \hat{q}_2 \frac{\partial p_2}{\partial q_1} \frac{d\hat{q}_1}{dw} = 0.
\] (6)

We know from (3) that \( \frac{d\hat{q}_2}{dw} < 0 \) and \( \frac{d\hat{q}_1}{dw} > 0 \). Therefore, given that \( \frac{\partial p_2}{\partial q_1} < 0 \), it is straightforward that \( (w^{*} - c_u) \) must be negative in order for (6) to be satisfied.

**Lemma 1.** Under observable contracting, the equilibrium input price is always lower than the upstream marginal cost, \( w^{*} < c_u \).

According to Lemma 1, the input price reflects a subsidy from \( U_2 \) to its respective downstream firm \( D_2 \). This finding, as well as its intuition, is in line with Milliou & Petrakis (2007), who consider the case where both vertical chains are separated. In our framework, the separated vertical chain, via a lower input price, can commit to a more aggressive behavior in the final-good market. The best-response curve of its downstream firm shifts out, resulting - since best-response curves are downward sloping - in lower final-good quantity for the rival integrated chain, and higher quantity and gross profits for the own downstream firm. The portion of these gross profits that is transferred upstream via the fixed fee, more than compensates the upstream firm for the subsidy it offers.

Before proceeding to the post-merger case, we should stress here that the finding in Lemma 1 remains robust under upstream cost asymmetry: the equilibrium input price will always be lower than \( c_{U_2} \) regardless of how the latter compares to \( c_{U_1} \).

### 3.2. The post-merger case
When the merger occurs, firms $I$ and $D2$ choose simultaneously and independently their final-good outputs to maximize profits:

$$\max_{q_i} \pi_I = (p_i(q_i, q_2) - c_{D1} - c_U)q_i + (w - c_U)q_2 + F,$$

$$\max_{q_2} \pi_{D2} = p_2(q_1, q_2)q_2 - (w + c_{D2})q_2 - F.$$

It is straightforward that the profit maximization problem of $D2$ is unaffected by the merger. The newly merged firm $I$ has now profits from two sources: the term $(p_i(q_1, q_2) - c_{D1} - c_U)q_i$ captures, as in the pre-merger case, profits from sales of the final good, whereas the term $(w - c_U)q_2 + F$ reflects profits from selling the input to the independent downstream rival $D2$. Since downstream competition is over quantities, however, firm $I$ cannot affect its sales of the input upstream by increasing sales of its downstream rival and thus its profit maximization problem in the downstream market also remains unaffected by the merger. Therefore, the last-stage subgame equilibrium final-good outputs and prices are the same as in the pre-merger case.

Next, we solve the second stage of the game, i.e., we determine the equilibrium contract terms. The newly merged firm $I$ uses the fixed fee to fully extract $D2$’s profits and thus set the input price so as to maximize,

$$\max_w (\hat{p}_1(w) - c_{D1} - c_U)\hat{q}_1(w) + (\hat{p}_2(w) - c_{D2} - c_U)\hat{q}_2(w)$$

Hence, the input price is actually chosen so as to maximize overall industry profits. The first order condition, after using (1) and (2), is given by:

$$\hat{q}_1 \frac{\partial p_1}{\partial q_2} \frac{d\hat{q}_2}{dw} + (w - c_U) \frac{d\hat{q}_2}{dw} + \hat{q}_2 \frac{\partial p_2}{\partial q_1} \frac{d\hat{q}_1}{dw} = 0$$

Compared to (6), expression (8) contains the additional term,

$$\hat{q}_1 \frac{\partial p_1}{\partial q_2} \frac{d\hat{q}_2}{dw} > 0.$$
with the positive sign stemming from the fact that \( \partial p / \partial q < 0 \) and \( d\hat{q} / dw < 0 \). Therefore, the upstream horizontal merger creates partial vertical foreclosure: any increase in the input price will decrease sales of \( D2 \) which will in turn increase the final-good price and the merged firm’s profits from downstream operations. The independent upstream firm \( U2 \) cannot internalize this effect and thus the input price will increase as a result of the merger, \( w^M > w^S \). The effect of the merger on consumer surplus is then clear-cut. Since the equilibrium input price increases, we know from (3) that both final-good prices will increase and total output will be reduced, causing a consumer surplus reduction.

**Proposition 1.** Under upstream cost symmetry and observable contracting, a horizontal merger between the vertically integrated firm and the independent upstream supplier always (i) increases the input price and (ii) decreases consumer surplus.

Proposition 1 provides support of the basic downstream antitrust concern about such mergers: a merger between the vertically integrated firm and the independent upstream supplier increases the input price and forces the independent downstream firm to adopt a less aggressive behavior, with obvious consequences for prices and consumer surplus. Our contribution is that we formally incorporate the vertical partial foreclosure effect, which is well-established in the literature on vertical mergers, into horizontal merger analysis.

Note at this point that, since in the pre-merger situation the integrated firm directs all its production to its subsidiary, the merchant input-market is a monopoly in the pre-merger situation and remains so after the merger. Hence the merger has no impact on the input-market concentration and all its consequences on prices and consumer surplus derive solely from its vertical partial foreclosure effect. It is straightforward that the latter effect is more pronounced the less differentiated final goods are.

Finally, we solve the first stage of the game by showing that the merger is always beneficial for the merging parties and the industry as a whole. Recall that in the pre-merger case the input price is chosen so as to maximize the unintegrated vertical chain’s profits (see (5)), rather than total industry profits. In the post-merger case, however, the input price is chosen so as to maximize overall industry profits (see (7)). Therefore, it must hold that \( \pi^{M^*}_{\text{ind}}(w^{M^*}) > \pi^{S^*}_{\text{ind}}(w^{S^*}) \). Since overall industry profits increase as a result of the merger, and \( D2 \)’s net profits remain unaffected (in both cases are equal to zero), it must hold that the
combined net profits of $U1-D1$ and $U2$ increase, implying that the merger is beneficial for the merging parties.\footnote{In light of the analysis in Section 5, note that the same reasoning applies to the case of unobservable contracting, i.e., when the vertically integrated firm does not observe the contract stipulated in the vertically separated chain.}

Before closing this section, it should be noted that the results in Proposition 1 remain qualitatively robust under the alternative assumption of downstream Bertrand competition.\footnote{However, Lemma 1 is no longer valid. Under downstream Bertrand competition, the independent upstream firm no longer wants to induce an aggressive behaviour in the downstream market since prices, unlike quantities, are strategic complements, and thus downstream production is not subsidized.} In fact, the merger’s negative impact on final-good prices will be more pronounced under downstream price competition than under downstream quantity competition since in the former case, besides the already identified vertical foreclosure effect, there is also a downstream accommodation effect: for any given input price, the newly merged firm realizes that any customer lost in the downstream market can be recovered via the upstream market providing it with a credible commitment to relax downstream competition.\footnote{This downstream accommodation effect was first identified by Chen (2001). See also Ordover & Shaffer (2007), Arya et al. (2008), Hoffler & Schmidt (2008) and Bourreau et al. (2011).}

4. Upstream cost asymmetry

In this section, we explore the role of efficiency gains associated with the merger. In particular, we address the following question: If the upstream merger increases efficiency in the merging firms, and given that one of the merging parties is a vertically integrated firm that, in the pre-merger case, uses all its own upstream production for its downstream division, is it possible that overall consumer surplus increases even though input prices increase and some consumers are worse off?

We modify our baseline model by introducing upstream cost asymmetry. We consider two cases: the independent upstream firm is less efficient than the upstream division of the vertically integrated firm and vice versa. We assume that, in the post-merger case, the more efficient firm transfers its technology to the less efficient firm: the newly merged firm operates with marginal costs $\hat{c}_u = \min\{c_{U1}, c_{U2}\}$. When $\hat{c}_u = c_{U1} < c_{U2}$, the merger creates efficiency gains in the upstream production that is directed to the independent downstream rival, whereas $c_{U1} > c_{U2} = \hat{c}_u$ implies efficiency gains in the upstream production directed to the downstream division of the merged firm.
For tractability reasons, we restrict attention to the following set of linear inverse demand functions (Singh & Vives, 1984),

\[ p_i = 1 - q_i - \theta q_j, \quad i, j = 1, 2, \quad i \neq j, \quad \theta \in (0,1) \]

where the inverse demand intercept, without loss of generality, is normalized to one and the parameter \( \theta \) measures the degree of product substitutability. The higher is \( \theta \), the closer substitutes final goods are. In this section only, we assume that \( c_{d1} = c_{d2} = 0 \), which, besides simplifying calculations, allows to focus on the effects of the merger on input prices and consumer surplus stemming solely from upstream cost differences. All proofs in this section are relegated to Appendix B.

4.1. The pre-merger case.

Firms \( U1-D1 \) and \( D2 \) choose simultaneously and independently their final-good outputs to maximize profits:

\[
\begin{align*}
\max_{q_i} \pi_{U1-D1} &= (1 - q_1 - \theta q_2 - c_{U1})q_1, \\
\max_{q_2} \pi_{D2} &= (1 - q_2 - \theta q_1 - w)q_2 - F.
\end{align*}
\]

The first order conditions give rise to the following best-response functions:

\[
\begin{align*}
q_1(q_2, c_{U1}) &= \frac{1 - c_{U1} - \theta q_2}{2}, \\
q_2(q_1, w) &= \frac{1 - w - \theta q_1}{2}.
\end{align*}
\]

Solving the system of best-response functions in (11), we obtain the last-stage subgame equilibrium outcomes as functions of \( w \) and \( c_{U1} \):
\[ q_1(w,c_{U1}) = \frac{(2-\theta) + \theta w - 2c_{U1}}{4-\theta^2}, \quad q_2(w,c_{U1}) = \frac{(2-\theta) + \theta c_{U1} - 2w}{4-\theta^2}, \]

\[ Q(w,c_{U1}) = q_1(w,c_{U1}) + q_2(w,c_{U1}) = \frac{2-w-c_{U1}}{2+\theta}, \quad (12) \]

\[ p_1(w,c_{U1}) = \frac{(2-\theta) + \theta w + (2-\theta^2)c_{U1}}{4-\theta^2}, \quad p_2(w,c_{U1}) = \frac{(2-\theta) + \theta c_{U1} + (2-\theta^2)w}{4-\theta^2}. \]

It can be easily checked that the above last-stage subgame equilibrium outcomes satisfy the properties described in (3).

The independent upstream firm \( U2 \) uses the fixed fee to fully extract \( D2 \)'s profits,

\[ F = [p_2(w,c_{U1}) - w]q_2(w,c_{U1}), \quad (13) \]

and thus sets the input price to maximize:

\[ \max_w \pi_{U2} = (w - c_{U2})q_2(w,c_{U1}) + F = [p_2(w,c_{U1}) - c_{U2}]q_2(w,c_{U1}). \quad (14) \]

From the first order condition of (14), we obtain the equilibrium input price:

\[ w^*(c_{U1},c_{U2}) = \frac{-(2-\theta)\theta^2 - c_{U1}\theta^3 + 2(4-\theta^2)c_{U2}}{4(2-\theta^2)} < c_{U2}, \quad (15) \]

which implies a subsidy from the independent upstream supplier to the independent downstream firm, in the spirit of Lemma 1.

4.2. The post-merger case

Firms \( I \) and \( D2 \) choose simultaneously and independently their final-good outputs to maximize profits:
\[
\max_{\pi} \pi = (1 - q_1 - \theta q_2 - \hat{c}_U) q_1 + (w - \hat{c}_U) q_2 + F,
\]
\[
\max_{q_2} \pi_{D2} = (1 - q_2 - \theta q_1 - w) q_2 - F.
\]

The first order conditions of the above maximization problems give rise to the following best-response functions:
\[
q_1(q_2, \hat{c}_U) = \frac{1 - \hat{c}_U - \theta q_2}{2}, \quad q_2(q_1, w) = \frac{1 - w - \theta q_1}{2}.
\]  
(16)

Solving the system of best-response functions in (16), we obtain the last-stage subgame equilibrium outcomes as functions of \(w\) and \(\hat{c}_U\):
\[
q_1(w, \hat{c}_U) = \frac{(2 - \theta) + \theta w - 2 \hat{c}_U}{4 - \theta^2}, \quad q_2(w, \hat{c}_U) = \frac{(2 - \theta) + \theta \hat{c}_U - 2 w}{4 - \theta^2},
\]
\[
Q(w, \hat{c}_U) = q_1(w, \hat{c}_U) + q_2(w, \hat{c}_U) = \frac{2 - w - \hat{c}_U}{2 + \theta},
\]  
(17)

In light of our subsequent analysis, we make the following two observations regarding the last-stage subgame equilibrium outcomes in the pre- and post-merger case(s). For any given level of the input price, the merger (i) does not affect downstream equilibrium outcomes when \(\hat{c}_U = c_{U1}\) and (ii) increases total output and decreases both final-good prices when \(\hat{c}_U = c_{U2}\).

The merged firm \(I\) uses the fixed fee to fully extract \(D2\)'s profits,
\[
F = [p_2(w, \hat{c}_U) - w] q_2(w, \hat{c}_U),
\]  
(18)

and thus sets the input price to maximize:
\[
\max_w [p_1(w, \hat{c}_U) - \hat{c}_U] q_1(w, \hat{c}_U) + [p_2(w, \hat{c}_U) - \hat{c}_U] q_2(w, \hat{c}_U)
\]  
(19)
From the first order condition of (19), we obtain the equilibrium input price:

\[
w^*(\hat{c}_U) = \frac{(2-\theta^2)\theta + \hat{c}_U(8 - 4\theta - 2\theta^2 - \theta^4)}{2(4 - 3\theta^2)} > \hat{c}_U. \tag{20}
\]

The expression in (20) implies that while an independent \( U_2 \) subsidizes the input purchases of \( D_2 \), in the post-merger case the equilibrium input price is always above the upstream marginal cost.

We consider first the case where the independent upstream firm is less efficient than the upstream division of the vertically integrated firm. The effects of the merger on input price, final-good prices and consumer surplus are summarized in the next Proposition.

**Proposition 2.** Under upstream cost asymmetry and observable contracting, when \( c_{U_1} < c_{U_2} \), a horizontal merger between the vertically integrated firm and the independent upstream supplier decreases the input and final-goods prices and increases consumer surplus if and only if

\[
\frac{1-c_{U_2}}{1-c_{U_1}} < \gamma_1(\theta) = \frac{8 - 4\theta - 4\theta^2 + \theta^3}{2(4 - 3\theta^2)}.
\]

When \( U_2 \) is less efficient than \( U_1 \), the merger creates efficiency gains that, while they lower the cost of the upstream production directed to the independent downstream rival, they do not affect the cost of the upstream production directed to the downstream division of the merged firm. This implies that the merger affects downstream equilibrium only through one channel, the input price. Concerning the merger’s impact on the latter, two effects are in work. By ending the subsidization of \( D_2 \), the merger tends to raise, *ceteris paribus*, the input price, which induces \( D_2 \) to behave less aggressively and thus pushes both final-good prices upwardly. At the same time, however, the merger creates efficiency gains in the supply of the input to the independent downstream firm causing the input price to fall. When these efficiency gains are sufficiently large to outweigh the former effect, the merger results to an input-price reduction, which causes both final-good prices to decrease thereby making all consumers better off.
Consider now the case where the independent upstream firm is more efficient than the upstream division of the vertically integrated firm. The effects of the merger on input price, final-good prices and consumer surplus are summarized in the next Proposition.

**Proposition 3.** Under upstream cost symmetry and observable contracting, when $c_{u_1} > c_{u_2}$, a horizontal merger between the vertically integrated firm and the independent upstream supplier:

(i) always increases the input price and the final-good price of the independent downstream firm,

(ii) decreases the final-good price of the vertically integrated firm if and only if

$$1 - \frac{c_{u_1}}{1 - c_{u_2}} < \gamma_2(\theta) = \frac{2[8 - 14\theta^2 + \theta^3 + 5\theta^4]}{(4 - 3\theta^2)^2},$$

(iii) increases consumer surplus if and only if

$$1 - \frac{c_{u_1}}{1 - c_{u_2}} < \gamma_3(\theta) = \frac{2[(2 - \theta^2)\sqrt{A} - \theta^3]}{16 - 20\theta^2 + 5\theta^4},$$

with $A = (64 - 64\theta - 96\theta^2 + 80\theta^3 + 52\theta^4 - 20\theta^5 - 15\theta^6)/(4 - 3\theta^2) > 0$ and $\gamma_3(\theta) < \gamma_2(\theta)$.

When the independent upstream firm is more efficient than the upstream division of the vertically integrated firm, on the one hand, the merger does not lower the cost of the upstream production directed to the independent downstream firm, leaving at play only the vertical foreclosure effect. Hence, as in the case of upstream cost symmetry, the input price always increases pulling with it the final-goods prices. On the other hand, the merger also creates efficiency gains in the upstream production directed to the downstream division of the merged firm, thus tending to decrease both final-good prices.

As it turns out, the final-good price of the independent downstream firm always increase due to the merger irrespective of the magnitude of the efficiency gains. However, when the efficiency gains are sufficiently large, the final-good price of the vertically integrated firm decreases and consumer surplus increases as a result of the merger; as indicated in Proposition 3, the potential decrease in the final-good price of the vertically integrated firm is a necessary but not sufficient condition for consumer surplus to increase.

Milliou & Pavlou (2013), in examining, among other things, the role of efficiency gains in upstream merger analysis, show that an upstream merger between vertically *separated* firms can increase consumer surplus as long as it reduces input prices, in which case all consumers
are better off. Our analysis reveals that when one of the merging parties is a vertically integrated firm overall consumer surplus may increase due to the merger even though the input price always increases and some consumers are worse off.

We make three final remarks regarding both cases of upstream cost asymmetry considered above. First, the merger’s positive effect on consumer surplus is more likely the more differentiated final goods are. This is so because the higher is the degree of product differentiation the weaker the vertical partial foreclosure effect is. In the extreme case where final goods are independent in demand, the vertical foreclosure effect vanishes and thus the merger always increases consumer surplus.\(^{13}\)

Second, the merger is always beneficial for the merging parties. Since even a merger between symmetric upstream firms is beneficial for the merging parties (see Section 3), a merger between asymmetric firms increases their profits even more, due to efficiency gains it creates, and this, irrespectively of whether these gains lower the input cost of the downstream division of the merged firm or the independent downstream rival.

Third, the results in Propositions 2 and 3 remain qualitatively robust under the assumption of downstream Bertrand competition. As discussed in Section 3, in the absence of any efficiency gains, the merger’s negative impact on consumer surplus is less pronounced under downstream quantity competition than under downstream price competition. Therefore, in the latter case, the efficiency gains entailed by the merger must be even larger to outweigh the vertical foreclosure effect.

5. Unobservable contracting

The analysis thus far suggests that the upstream merger is detrimental to consumers unless it generates significant efficiency gains. In this section, we explore the possibility that the merger increases consumer surplus even in the absence of any efficiency gains. To this end, we return to the case of upstream cost symmetry, however, we now assume that in the pre-merger situation, the contract stipulated in the vertically separated chain is unobservable by

\[^{13}\text{Note that when products are totally differentiated, i.e., } \theta = 0 \text{, from Propositions 2 and 3, we have that } \gamma_f(0) = 1 \text{ and } \gamma_f(0) = 1. \text{ The former implies that the merger increases consumer surplus when } (1-c_{u_2})/(1-c_{u_1}) < 1 \text{ which is always true given that } c_{u_1} < c_{u_2}, \text{ whereas the latter implies that the merger increases consumer surplus when } (1-c_{u_2})/(1-c_{u_1}) < 1 \text{ which is also always true given that } c_{u_1} > c_{u_2}.\]
the vertically integrated firm. Observable contracts have a commitment value in the sense that they can strategically affect the rivals’ behavior. This strategic commitment is no longer possible under unobservable contracts.

The pre-merger equilibrium is determined as follows (all proofs in this section are relegated to Appendix C). From (1), we obtain the best-response function of the downstream division of the integrated firm $q_i(q_z)$. Given contract unobservability, the integrated firm’s best-response function in the downstream market does not depend on the input price established by the independent upstream firm. Accordingly, from (2), we obtain the independent downstream firm’s best-response $q_z(q_i, w)$. The associated final-good prices for the integrated firm and $D_2$ are given, respectively, by $p_1(q_i(q_z), q_z)$ and $p_2(q_z(q_i, w), q_z)$. The independent upstream firm $U_2$ uses the fixed fee to fully extract $D_2$’s profits,

$$F = [p_2(q_z(q_i, w), q_i) - w - c_{D_2}]q_z(q_i, w),$$

and thus sets the input price so as to maximize,

$$\max_w \pi_{U_2} = (w - c_U)q_z(q_i, w) + F = [p_2(q_z(q_i, w), q_i) - c_U - c_{D_2}]q_z(q_i, w).$$

The first order condition of the above maximization problem, after using (2), is given by:

$$(w - c_U) \frac{dq_z}{dw} = 0.$$

It is straightforward that in order for (23) to be satisfied it must hold that $\tilde{w}^{*\prime} = c_U$.

**Lemma 2.** Under unobservable contracting, the equilibrium input price in the pre-merger case is always equal to the upstream marginal cost, $\tilde{w}^{*\prime} = c_U$.

---

14 Whereas in some industries the assumption of observable contracts seems quite reasonable, in others it is not very plausible since contracts are kept highly confidential. An important strand in the literature on secret vertical contracting considers one upstream manufacturer selling its product to many downstream firms (see, e.g., Hart & Tirole, 1990; O’Brien & Shaffer, 1992; Rey & Vergé, 2004). In such setting, the equilibrium contracts depend on the nature of the downstream firms’ out-of-equilibrium beliefs. Since in our model one upstream firm contracts with only one downstream firm, out-of-equilibrium beliefs play no role.
Irmen (1998), Fumagalli & Motta (2001) and Symeonidis (2010) consider a setting with two competing vertically separated chains and analyse, among other things, the case of unobservable two-part tariff contracts. Contract unobservability eliminates any strategic effect associated with the choice of input prices: for any given input price charged by one upstream firm, the best reply of the other upstream firm is to set an input price equal to upstream marginal cost and use the fixed fee to get profit. In other words, the two-part tariff contract has no pre-commitment effect and thus each upstream firm is indifferent between stipulating an exclusive contract with a downstream firm and vertically integrating. Clearly, this insight also applies to the case where one vertical chain is already vertically integrated.

Moreover, as in the case of observable contracting, the finding in Lemma 1 remains robust under upstream cost asymmetry.

It is straightforward by construction of the model that there is no issue regarding contract observability in the post-merger case. Therefore, the equilibrium analysis in subsection 3.2 is still valid and the post-merger equilibrium input price must satisfy (8). Unfortunately, under a general inverse demand function we cannot determine whether that equilibrium input price will be lower or higher than upstream marginal cost and consequently we cannot say whether the merger will increase or decrease the input price. Nevertheless, by restricting attention to the linear demand function given in (9), we are able to obtain the following result.

**Lemma 3.** Under upstream cost symmetry, unobservable contracting and a linear demand function, the equilibrium input price in the post-merger case is higher (lower) than the upstream marginal cost whenever \( \alpha < \gamma(\theta) \) (\( \alpha > \gamma(\theta) \)), with

\[
\alpha = \frac{1 - c_{U} - c_{D2}}{1 - c_{U} - c_{D1}} \quad \text{and} \quad \gamma(\theta) = \frac{4 + \theta^2}{4\theta}.
\]

Note that for \( 0 < \theta < 1 \) it holds that \( \gamma(\theta) > 1 \). On the one hand, whenever \( c_{D1} \leq c_{D2} \) holds, the inequality \( \alpha < \gamma(\theta) \) is always satisfied: a sufficient condition for the equilibrium input price to be higher than upstream marginal cost is that the downstream division of firm I is more cost-efficient than (or equally cost-efficient to) \( D2 \). On the other hand, in order for the inequality \( \alpha > \gamma(\theta) \) to be satisfied it must be the case that \( c_{D1} > c_{D2} \): a necessary condition for the equilibrium input price to be lower than upstream marginal cost is that the downstream division of firm I is less cost-efficient than \( D2 \). The lower (higher) is the cost-difference.
\( c_{d_1} - c_{d_2} > 0 \), and thus the lower (higher) is the parameter \( a \), the less (more) likely is that the condition \( a > \gamma(\theta) \) be satisfied.

Recall that, in the post-merger case, the equilibrium input price is chosen so as to maximize total industry profits. Suppose that there is downstream cost symmetry, i.e., \( c_{d_1} = c_{d_2} \). Since downstream firms impose a negative externality upon each other, the merged firm will set an input price above marginal cost in order to correct for this externality. When there is downstream cost asymmetry, the input price will be further adjusted in order for sales of final-good to be shifted to the more cost-efficient, and thus more profitable, downstream firm. When \( c_{d_1} < c_{d_2} \), the merged firm has an incentive to further increase the input price above upstream marginal cost to shift final-good sales to its more cost-efficient downstream division. When \( c_{d_1} > c_{d_2} \), the merged firm has an incentive to decrease the input price to shift final-good sales to the more cost-efficient downstream rival. If the degree of downstream cost asymmetry is high enough, it then becomes optimal for the merged firm to set an input price below marginal cost.

From Lemmata 2 and 3, it is straightforward that the merger decreases the input price when \( a > \gamma(\theta) \), and so it decreases final-good prices and increases consumer surplus.

**Proposition 4.** Under upstream cost symmetry, unobservable contracting and a linear demand function, a horizontal merger between the vertically integrated firm and the independent upstream supplier (i) decreases (increases) the input price and (ii) increases (decreases) consumer surplus whenever \( a > \gamma(\theta) \) (\( a < \gamma(\theta) \)), with \( a \) and \( \gamma(\theta) \) given in Lemma 3.

Proposition 4 highlights the fact that the upstream merger may increase consumer surplus even in the absence of exogenous cost-synergies between the merging firms. By considering the case of two-part tariff contracts and downstream Cournot competition, Milliou & Petrakis (2007) show, among other things, that an upstream merger between two vertically separated firms always decreases the input price and increases consumer surplus, however, it is never profitable for the merging parties. In our context, where one of the merging parties is a vertically integrated firm, the merger is always profitable (see the discussion in Section 3 and especially footnote 10) thus allowing us to provide a theoretical explanation of observed upstream mergers that might be beneficial for consumers even when they do not generate efficiency gains in upstream production.
Finally, it should be noted here that Proposition 4 does not remain robust under the alternative assumption of downstream Bertrand competition. In the pre-merger case, as in the case of downstream Cournot competition, there is upstream marginal cost pricing in the vertically separated chain due to the presence of fixed fees (Lemma 2 remains valid). In the post-merger case, unlike the case of downstream Cournot competition, the input price will never be lower than upstream marginal cost (Lemma 3 is no longer valid): under downstream price competition, it is less urgent for the merged firm to induce an aggressive behavior in the downstream market since prices, unlike quantities, are strategic complements, and thus downstream production is never subsidized. Therefore, under unobservable contracting, the effects of the merger on consumer surplus crucially depend on the mode of downstream competition.

6. Conclusions

We have studied upstream horizontal mergers when one of the merging parties is a vertically integrated firm. We have considered a two-tier market consisting of two competing vertical chains, with one upstream and one downstream firm in each chain, assuming that one vertical chain is vertically integrated whereas the other chain is vertically separated. We have also assumed downstream Cournot competition and that firms in the vertically separated chain trade through a two-part tariff contract. The contract stipulated in the separated chain can be either observable or unobservable by the integrated chain (firm).

Under upstream cost symmetry and observable contracting, we have shown that a horizontal merger between the vertically integrated firm and the independent input supplier raises input price and induces a less aggressive behavior of the remaining independent downstream producer, shifting final-good sales towards the downstream affiliate of the integrated firm. The higher input price ultimately induces a rise in the final-good prices and fall in total output, thus making consumers worse off. Since in the pre-merger situation the vertically integrated firm sells no input in the merchant market, the latter is, and remains post-merger, a monopoly. Hence, the merger does not affect concentration in the upstream merchant market, and its negative impact on consumer surplus stems solely from a vertical partial foreclosure effect.

We have also identified two cases under which consumer surplus may increase due to the merger. In the first case, there is observable contracting but upstream costs are asymmetric.
We have assumed that, in the post-merger situation, the more efficient firm transfers its technology to the less efficient firm so that the merger generates efficiency gains. We have shown that overall consumer surplus may increase due to the merger even though the input price always increases and some consumers are worse off. In the second case, upstream costs are symmetric but there is unobservable contracting in the pre-merger situation. We have demonstrated that the input price may decrease and consumer surplus may increase as a result of the merger even in the absence of exogenous cost-synergies between the merging firms.

In all cases under consideration, the upstream merger is always profitable for the merging parties. In contrast to the literature on upstream mergers in vertically separated industries, two key insights from our analysis is that upstream mergers can be profitable and beneficial to consumers even in the absence of any efficiency gains, and once efficiency gains are taken into account, a reduction in input price is not always a necessary condition for an increase in consumer surplus.

While our formal analysis is based on take-it-or-leave-it offers, all our results extend straightforwardly to the situation where the independent upstream firm (in the pre-merger case) or the merged firm (in the post-merger case) engage in Nash bargaining with the independent downstream firm. As is well known, under two-part tariff contracts, the Nash bargaining solution can be found in two steps. First, the bargaining pair chooses the input price in order to maximize its joint surplus, which implies that the equilibrium input prices obtained in the previous sections are still valid. Second, firms negotiate the fixed fees in order to divide their maximized joint surplus. While bargaining implies different equilibrium fixed fees with the independent downstream firm no longer making zero net profits, fixed fees are simply a device used to transfer surplus and have no impact on marginal costs or quantities produced. Hence, the merger’s effect on final-good prices, quantities and consumer surplus - for all cases under consideration - remains under bargaining the same as under a take-it-or-leave-it offer.

Future research may consider alternative industry settings with a larger number of competing vertical chains (both integrated and separated) and/or non-exclusive relations between upstream and downstream firms (the latter will allow for the integrated firm’s participation in merchant input market, either as a seller, or even as a strategic buyer), in order to examine the impact of such type of mergers under different market structures.

Appendix A. Upstream cost symmetry and observable contracting
As noted in subsection 3.1, the last-stage subgame equilibrium final-good outputs and prices as functions of the input price are given by: \( \hat{q}_1(w) \), \( \hat{q}_2(w) \), \( \hat{p}_1(w) = p_1[\hat{q}_1(w), \hat{q}_2(w)] \) and \( \hat{p}_2(w) = p_2[\hat{q}_1(w), \hat{q}_2(w)] \). As also noted in subsection 3.2, these equilibrium outcomes are the same regardless of whether the merger occurs or not. We derive here their properties described in (3).

Note first that \( \hat{q}_1 \) depends on \( w \) only indirectly through \( \hat{q}_2 \) so that \( \hat{q}_1(w) = q_1[\hat{q}_2(w)] \) and \( \frac{d\hat{q}_1(w)}{dw} = (\frac{dq_1}{dq_2})(\frac{d\hat{q}_2(w)}{dw}) \). Given strategic substitutability (see Assumption 2) it holds that \( \frac{dq_1}{dq_2} < 0 \). It is then straightforward that \( \frac{d\hat{q}_1(w)}{dw} \) and \( \frac{d\hat{q}_2(w)}{dw} \) have opposite signs.

We next show that \( \frac{d\hat{q}_2(w)}{dw} < 0 \).

The last-stage subgame equilibrium final-good outputs \( \hat{q}_1(w) \) and \( \hat{q}_2(w) \) must satisfy the first-order conditions in the downstream market, therefore (2) can be written as:

\[
p_2[\hat{q}_1(w), \hat{q}_2(w)] + \hat{q}_2(w) \frac{\partial p_2}{\partial q_2} - w - c_2 = 0.
\]

Using the implicit function theorem in the above expression, we obtain:

\[
\frac{d\hat{q}_2(w)}{dw} = \frac{1}{\frac{\partial p_2}{\partial q_2} \hat{q}_2 \frac{dq_1}{dq_2} + \frac{\partial p_2}{\partial q_1} \frac{dq_1}{dq_2} - \frac{\partial^2 \pi_{D2}}{\partial q_1^2}} < 0,
\]

where the denominator \( \frac{\partial^2 \pi_{D2}}{\partial q_2^2} \) is negative due to Assumption 1. Therefore, it holds that \( \frac{d\hat{q}_2(w)}{dw} < 0 \) and \( \frac{d\hat{q}_1(w)}{dw} > 0 \). Moreover, given that \( |\frac{dq_1}{dq_2}| < 1 \), it also holds that \( \frac{d\hat{q}_1(w)}{dw} < |\frac{d\hat{q}_2(w)}{dw}| \). The last inequality implies that an increase in the input price decreases the total quantity supplied in the downstream market, i.e., \( \frac{dQ(w)}{dw} < 0 \).

Regarding the effect of \( w \) on \( \hat{p}_2 \), we have that,

\[
\frac{d\hat{p}_2(w)}{dw} = \frac{\partial p_2}{\partial q_2} \frac{d\hat{q}_2(w)}{dw} + \frac{\partial p_2}{\partial q_1} \frac{d\hat{q}_1(w)}{dw} = \frac{\partial p_2}{\partial q_2} \frac{d\hat{q}_2(w)}{dw} + \frac{\partial p_2}{\partial q_1} \frac{d\hat{q}_1(w)}{dw} =
\]

\[
= \frac{d\hat{q}_2(w)}{dw} \left[ \frac{\partial p_2}{\partial q_2} + \frac{\partial p_2}{\partial q_1} \frac{dq_1}{dq_2} \right] > 0,
\]

where the bracketed term in the last inequality is negative since \( |\frac{\partial p_2}{\partial q_2}| > |\frac{\partial p_2}{\partial q_1}| \) and \( |\frac{dq_1}{dq_2}| < 1 \).
Finally, regarding the effect of $w$ on $\hat{p}_1$, we have that,

$$\frac{d\hat{p}_1(w)}{dw} = \frac{d\hat{q}_2(w)}{dw} \left[ \frac{\partial p_1}{\partial q_2} + \frac{\partial p_1}{\partial q_1} \frac{dq_1}{dq_2} \right] > 0.$$ 

An increase in $w$ affects indirectly $\hat{p}_1$ through $\hat{q}_2$ in two ways: On the one hand, a decrease in $\hat{q}_2$ increases $\hat{p}_1$ - a second order effect. On the other hand, a decrease in $\hat{q}_2$ leads to an increase in $\hat{q}_1$ which in turn decreases $\hat{p}_1$ - a third order effect. It is natural to assume that the second order effect is of greater importance than the third order effect implying that $\hat{p}_1$ increases with $w$.

Appendix B. Upstream cost asymmetry

We first characterize the final equilibrium outcomes. In the pre-merger case, equilibrium outcomes are as follows:

$$q_1^{\ast\ast}(c_{U_1}, c_{U_2}) = [p_1^{\ast\ast}(c_{U_1}, c_{U_2}) - c_{U_1}] = \frac{(4 - 2\theta - \theta^2) + 2\theta c_{U_2} - (4 - \theta^2)c_{U_1}}{4(2 - \theta^2)},$$

$$q_2^{\ast\ast}(c_{U_1}, c_{U_2}) = [p_2^{\ast\ast}(c_{U_1}, c_{U_2}) - w^{\ast\ast}(c_{U_1}, c_{U_2})] = \frac{(2 - \theta) + \theta c_{U_1} - 2c_{U_2}}{2(2 - \theta^2)},$$

$$CS^{\ast\ast}(c_{U_1}, c_{U_2}) = \frac{(16 - 20\theta^2 + 5\theta^4)(1 - c_{U_1})^2 + 4(4 - 3\theta^2)(1 - c_{U_2})^2 + 4\theta^3(1 - c_{U_1})(1 - c_{U_2})}{32(2 - \theta^2)^2}.$$

In the post-merger case, equilibrium outcomes are as follows:

$$q_1^{\ast\ast}(\hat{c}_U) = [p_1^{\ast\ast}(\hat{c}_U) - \hat{c}_U] = \frac{(4 - 2\theta - \theta^2)(1 - \hat{c}_U)}{2(4 - 3\theta^2)},$$

$$q_2^{\ast\ast}(\hat{c}_U) = [p_2^{\ast\ast}(\hat{c}_U) - w^{\ast\ast}(\hat{c}_U)] = \frac{2(1 - \theta)(1 - \hat{c}_U)}{4 - 3\theta^2},$$

$$CS^{\ast\ast}(\hat{c}_U) = \frac{(8 - 4\theta - 3\theta^2)(1 - \hat{c}_U)^2}{8(4 - 3\theta^2)}.$$

26
B.1. The case of $c_U = c_{U1} < c_{U2}$.

First, we derive the condition under which final-good quantities are positive under both the pre- and post-merger case(s), and then show that $w^{sr}(c_{U1}, c_{U2}) < c_{U2}$ and $w^{ur}(c_{U1}) > c_{U1}$.

It is straightforward that both $q^{mr}(c_{U1})$ and $q^{ur}(c_{U1})$ are positive whenever $c_{U1} < 1$. The requirement that $q^{sr}(c_{U1}, c_{U2}) > 0$ reduces to:

$$c_{U1}(c_{U2}, \theta) \equiv \frac{2c_{U2} - (2 - \theta)}{\theta} < c_{U1}. \tag{B3}$$

Given the assumption that $c_{U1} < c_{U2}$, it must hold that $c_{U1}(c_{U2}, \theta) < c_{U2}$. It is straightforward that the latter condition is always true whenever $c_{U2} < 1$. Therefore, condition (B3) can be written as,

$$\frac{2c_{U2} - (2 - \theta)}{\theta} < c_{U1} < c_{U2} < 1, \tag{B4}$$

or, rearranging the terms in the first inequality, as

$$\frac{1 - c_{U2}}{1 - c_{U1}} > f_1(\theta) = \frac{\theta}{2}. \tag{B5}$$

The requirement that $q_{U1}^{sr}(c_{U1}, c_{U2}) > 0$ reduces to $c_{U1} < [(4 - 2\theta - \theta^2) + 2\theta c_{U2}]/4 - \theta^2$, which is always true since for $c_{U2} < 1$ it holds that $c_{U2} < [(4 - 2\theta - \theta^2) + 2\theta c_{U2}]/4 - \theta^2$. Therefore, condition (B4) or (B5) guarantees that final-good quantities are positive under both the pre- and post-merger case(s).

Using (15), the requirement that $w^{sr}(c_{U1}, c_{U2}) < c_{U2}$ reduces to $[2c_{U2} - (2 - \theta)]/\theta < c_{U1}$, which is always true given (B4). Similarly, using (20), the requirement that $w^{ur}(c_{U1}) > c_{U1}$ reduces to $c_{U1} < 1$, which is always true given (B4).
Proof of Proposition 2. We define $\Delta w = w^s(c_{U1}, c_{U2}) - w^{Mr}(c_{U1})$. Using (15) and (20), we obtain:

$$\Delta w = \frac{(4 - \theta^2)[(1 - c_{U1})(8 - 4\theta - 4\theta^2 + \theta^3) - 2(4 - 3\theta^2)(1 - c_{U2})]}{4(4 - 3\theta^2)(2 - \theta^2)}.$$ 

It is straightforward that $\Delta w > 0$ whenever the bracketed term in the numerator of the above expression is positive, which yields,

$$\frac{1 - c_{U2}}{1 - c_{U1}} < \gamma_1(\theta) = \frac{8 - 4\theta - 4\theta^2 + \theta^3}{2(4 - 3\theta^2)}. \quad \text{(B6)}$$

It can be easily checked that $\gamma_1(\theta) > \gamma_1(\theta)$ which implies that the results in Proposition 2 hold when both firms are active in the downstream market.

Given that for any given level of input prices the downstream equilibrium outcomes are the same in both the pre- and post-merger cases, it is straightforward that the merger’s overall effect on final-good prices, total output and consumer surplus is solely determined by its effect on the input price. Therefore, whenever condition (B6) holds, both final-good prices decrease and total output increases, implying an increase in consumer surplus.

B.2. The case of $c_{U1} > c_{U2} = \hat{c}_U$.

First, we derive the condition under which final-good quantities are positive under both the pre- and post-merger case(s), and then show that $w^s(c_{U1}, c_{U2}) < c_{U2}$ and $w^{Mr}(c_{U2}) > c_{U2}$.

It is straightforward that both $q_1^{Mr}(c_{U2})$ and $q_2^{Mr}(c_{U2})$ are positive whenever $c_{U2} < 1$. The requirement that $q_1^{s}(c_{U1}, c_{U2}) > 0$ reduces to:

$$c_{U2}(c_{U1}, \theta) = \frac{(4 - \theta^2)c_{U1} - (4 - 2\theta - \theta^2)}{2\theta} < c_{U2}. \quad \text{(B7)}$$
Given the assumption that $c_{u_1} > c_{u_2}$, it must hold that $c_{u_2}(c_{u_1}, \theta) < c_{u_1}$. It is straightforward that the latter condition is always true whenever $c_{u_1} < 1$. Therefore, condition (B7) can be written as,

$$\frac{(4 - \theta^2)c_{u_1} - (4 - 2\theta - \theta^2)}{2\theta} < c_{u_2} < c_{u_1} < 1,$$  \hspace{1cm} (B8)

or, rearranging the terms in the first inequality, as

$$\frac{1 - c_{u_1}}{1 - c_{u_2}} > \bar{y}_2(\theta) = \frac{2\theta}{4 - \theta^3}.$$  \hspace{1cm} (B9)

The requirement that $q_2^*(c_{u_1}, c_{u_2}) > 0$ reduces to $c_{u_2} < [(2 - \theta) + \theta c_{u_1}] / 2$, which is always true since for $c_{u_1} < 1$ it holds that $c_{u_1} < [(2 - \theta) + \theta c_{u_1}] / 2$. Therefore, condition (B8) or (B9) guarantees that final-good quantities are positive under both the pre- and post-merger case(s).

Using (15), the requirement that $w^*(c_{u_1}, c_{u_2}) < c_{u_2}$ reduces to $[(4 - \theta^2)c_{u_1} - (4 - 2\theta - \theta^2)]/2\theta < c_{u_2}$, which is always true given (B8). Similarly, using (20), the requirement that $w^M(c_{u_2}) > c_{u_2}$ reduces to $c_{u_2} < 1$, which is always true given (B8).

**Proof of Proposition 3.** (i) Given that $w^*(c_{u_1}, c_{u_2}) < c_{u_2}$ and $w^M(c_{u_2}) > c_{u_2}$, it is straightforward that $w^*(c_{u_1}, c_{u_2}) < w^M(c_{u_2})$.

(ii) Using (15), (20), (B1) and (B2), we have that:

$$p_1^M(c_{u_2}) - p_1^S(c_{u_1}, c_{u_2}) = \frac{(4 - 3\theta^2)(1-c_{u_1}) - 2(1-c_{u_2})(8 - 14\theta^2 + \theta^3 + 5\theta^4)}{4(4 - 3\theta^2)(2 - \theta^2)},$$  \hspace{1cm} (B10)

and

$$p_2^M(c_{u_2}) - p_2^S(c_{u_1}, c_{u_2}) = \frac{\theta[(4 - 3\theta^2)(1-c_{u_1}) - 2\theta(1-\theta)(1-c_{u_2})]}{4(4 - 3\theta^2)}.$$  \hspace{1cm} (B11)
First, we derive the condition under which \( p_1^{M^*}(c_{U2}) < p_1^{S^*}(c_{U1}, c_{U2}) \) and then show that \( p_2^{M^*}(c_{U2}) > p_2^{S^*}(c_{U1}, c_{U2}) \). The expression in (B10) is negative whenever its numerator is negative, i.e.,

\[
(4 - 3\theta^2)(1 - c_{U1}) - 2(1 - c_{U2})(8 - 14\theta^2 + \theta^3 + 5\theta^4) < 0,
\]

which yields,

\[
\frac{1 - c_{U1}}{1 - c_{U2}} < \gamma_2'(\theta) = \frac{2[8 - 14\theta^2 + \theta^3 + 5\theta^4]}{(4 - 3\theta^2)^3}.
\]

After some straightforward calculations, it can be easily checked that the above expression is always true given (B9). The expression in (B11) is positive whenever the bracketed term in its numerator is positive, i.e.,

\[
(4 - 3\theta^2)(1 - c_{U1}) - 2\theta(1 - \theta)(1 - c_{U2}) > 0,
\]

yielding,

\[
\frac{1 - c_{U1}}{1 - c_{U2}} > \frac{2\theta(1 - \theta)}{(4 - 3\theta^2)},
\]

which is always true given (B9).

(iii) Regarding consumer surplus, we define \( \Delta CS = CS^{M^*}(c_{U2}) - CS^{S^*}(c_{U1}, c_{U2}) \). Using (B1) and (B2), and solving \( \Delta CS = 0 \) for \( 1 - c_{U1} \) we obtain two roots:

\[
(1 - c_{U1})_1 = \frac{2(1 - c_{U2})[(2 - \theta)^2\sqrt{A} - \theta^3]}{16 - 20\theta^2 + 5\theta^4} \quad \text{and} \quad (1 - c_{U1})_2 = -\frac{2(1 - c_{U2})[(2 - \theta)^2\sqrt{A} + \theta^3]}{16 - 20\theta^2 + 5\theta^4},
\]

with \( A = (64 - 64\theta - 96\theta^2 + 80\theta^3 + 52\theta^4 - 20\theta^5 - 15\theta^6)/(4 - 3\theta^2) > 0 \). Since we require that \( c_{U1} < 1 \), we disregard the second root since it is always negative. From the first root, we obtain that \( \Delta CS > 0 \) whenever.
After some tedious but straightforward calculations, it can be shown that \( \bar{\gamma}_2(\theta) < \gamma_3(\theta) < \gamma_2(\theta) \), which implies that (i) a necessary (but not sufficient) condition for consumer surplus to increase is that \( p_1 \) falls and (ii) the results in Proposition 3 hold when both firms are active in the downstream market.

Appendix C. Unobservable contracting

We first characterize the final equilibrium outcomes in both the pre- and post-merger case(s). In the pre-merger case, the equilibrium input price is given by \( w^* = c_U \) (see Lemma 2). Using (9), the final equilibrium outputs and consumer surplus are given by:

\[
\dot{q}_1^* = \frac{2(1-c_U-c_{D1}) - \theta(1-c_U-c_{D2})}{4-\theta^2}, \quad \dot{q}_2^* = \frac{2(1-c_U-c_{D2}) - \theta(1-c_U-c_{D1})}{4-\theta^2},
\]

\[
\hat{CS}^* = \frac{(1-c_U-c_{D1})^2(16-20\theta^2+5\theta^4) + 4(1-c_U-c_{D2})^2(4-3\theta^2) + 2(1-c_U-c_{D1})(1-c_U-c_{D2})\theta^3}{2(4-\theta^2)^2}
\]

(C1)

As discussed in the main text, there is no issue regarding contract observability in the post-merger case. Therefore, the equilibrium analysis in subsection 3.2 is still valid and the post-merger equilibrium input price must satisfy (8). Using (9), we obtain from (8) the equilibrium input price in the post-merger case as

\[
w^{M*} - c_U = \frac{\theta[(4+\theta)(1-c_U-c_{D1}) - 4\theta(1-c_U-c_{D2})]}{2(4-3\theta^2)}.
\]

(C2)

Using (C2), the final equilibrium outputs and consumer surplus are given by:
We make the following assumption:

**Assumption 4.** $\theta < \alpha < \frac{4-\theta^2}{2\theta}$ with \( \alpha = \frac{1-c_U-c_{D2}}{1-c_U-c_{D1}} \).

Assumption 4 guarantees that both firms will produce a positive quantity of the final-good for all cases under consideration.

**Proof of Lemma 3.** The equilibrium input price is higher (lower) than upstream marginal cost whenever the bracketed term in the RHS of (C2) is positive (negative). It is then straightforward that $w^{**} > c_U$ whenever $\alpha < \gamma(\theta) = (4 + \theta^2)/4\theta$ and vice versa.

**Proof of Proposition 4.** (i) The fact that the merger decreases (increases) the input price whenever $\alpha > \gamma(\theta)$ (\( \alpha < \gamma(\theta) \)) stems straightforwardly from Lemmata 2 and 3.

(ii) From (C1) and (C2), we calculate the following expression:

\[
CS^{**} - \tilde{CS}^{**} = \frac{\theta[(4+\theta^2)(1-c_U-c_{D1})-4\theta(1-c_U-c_{D2})]A}{8(4-3\theta^2)(4-\theta^2)^2},
\]

where $A = (4-3\theta^2)\theta(1-c_U-c_{D1})-2(8-5\theta^2)(1-c_U-c_{D2})$. After some straightforward but tedious calculations, it can be verified that, given Assumption 4, it holds that $A < 0$. Therefore, the sign of $CS^{**} - \tilde{CS}^{**}$ has the opposite sign of the bracketed term in the RHS of (C5). It is then straightforward that $CS^{**} > \tilde{CS}^{**}$ whenever $\alpha < \gamma(\theta) = (4 + \theta^2)/4\theta$ and vice versa.

**References**


