Does the US international debt affect the euro/dollar exchange rate?

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By

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ABSTRACT
The impact of the US international debt on the euro/dollar exchange rate is examined in the context of an Error Correction monetary model with rational expectations. Overall, the relative real income is the most economically significant determinant, whereas the debt is the most statistically significant determinant.

Keywords: Monetary model, US international debt, co-integration analysis, error correction model
JEL classification: F31

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1. Introduction

Since euro began its life in January 1999, it was trading at 1,167 dollars. The fall of the euro started immediately. By September 2000 the euro had fallen below 0,90 dollars and had reached its nadir of 0,83 dollars by early November. Thereafter, the euro fluctuated significantly with an upward trend. By the end of 2002, the euro was trading at par with the dollar. The appreciation of the euro continued and at the time of this writing (May 2008), the dollar price of the euro has hit a lifetime high, rising above 1,50 dollars. Figure 1 plots the euro/dollar exchange rate and the US federal government debt held by foreign and international investors. During the phase of depreciation, the debt declined by 17,38%, that is from $1,270 billion in the first quarter of 1999 to $1,050 billion in the last quarter of 2001. On the other hand, during the phase of appreciation, the debt increased by 108%, that is from $1,067 billion in the first quarter of 2002 to $2,200 billion in the first quarter of 2007. The correlation coefficient between the levels of the two variables is 0,97, while the correlation coefficient between the growth rates of the two variables is 0,54. Despite the tremendous increase in the US international debt, its potential effect on the euro/dollar has not been yet analyzed. The empirical literature, so far, has documented evidence in favor of a number of other fundamentals in determining the euro/dollar exchange rate (Sartore et al, 2002; Jamaleh, 2002; Ehrmann and Fratzscher, 2004; Brooks et al, 2004; Karfakis, 2006). Therefore, it is the purpose of this study to fill
in the gap in the literature by examining the link between the US international debt and the euro/dollar exchange rate. In order to frame our analysis, a monetary model with risk premia is set up and is solved under the assumption of weak rationality. The reduced-form of the model is an Error Correction (EC) representation, which relates the growth rate of the exchange rate to the growth rate of the monetary fundamentals and the growth rate of the debt. This is constitutes the theoretical contribution of the present study, since an EC exchange rate model is derived from a rational expectations framework.

Figure 1: The euro/dollar exchange rate and the US international debt

![Graph showing the relationship between the euro/dollar exchange rate and the US international debt. The graph displays data from 1999Q1 to 2006Q3, with exchange rates and debt levels plotted over time.]
2. Theoretical Model

The monetary model of the exchange rate determination is given by the following equations:

\[ m_t = p_t + y_t - ai_t \]  
\[ m^*_t = p^*_t + y^*_t - a^*i_t \]

\[ i^*_t - i_t = \Delta e_{t+1}^* |\Omega_t + \xi(d_t) \]

\[ e_t = p^*_t - p_t \]

where \( e_t \) is the logarithm of the exchange rate, defined as dollars per euro, \( e^*_{t+1} \) is the logarithm of the expected exchange rate prevailing at time \( t+1 \), with expectations formed at time \( t \), \( \Omega \) is the information set, \( m_t \) and \( m^*_t \) are the logarithms of the domestic and foreign money stocks, \( y_t \) and \( y^*_t \) are the logarithms of the domestic and foreign real income, \( p_t \) and \( p^*_t \) are the logarithms of domestic and foreign price levels, \( i_t \) and \( i^*_t \) are the short term domestic and foreign interest rates and \( d_t \) is the logarithm of the US federal government debt held by international investors. The domestic and foreign variables refer to Euro area and US, respectively.

The first two equations describe the equilibrium condition in the domestic and foreign money market. The term \( \alpha \) captures substitutability between the money and domestic bonds. Equation (3) describes the uncovered interest parity condition augmented with a risk premium \( \xi \) which depends positively on the level of the US international debt. As
debt increases, people are expecting that the US administration will inflate the economy in order to reduce the real value of the debt. The expected increase in the rate of inflation pushes interest rates up and the dollar down. This view is consistent with the monetary model. On the other hand, in the context of the portfolio balance model, an increase in risk premium on US assets, will increase interest rates and will depreciate the dollar in order to persuade the international investors to accumulate the higher stock of US debt.

Equation (4) describes the purchasing power parity doctrine.

Combining equations (1) to (4), yields:

\[ e_t = \phi' \zeta_t + \Theta e_{t+1} + \nu_t \]  

(5)

where \( \phi' = [\phi_1, \phi_2, \phi_3] \), \( \zeta_t = [(m_t^*, \mu_t, y_t, y_t^*, \delta_t)] \), \( \phi_1 = \alpha/(1+\alpha) \), \( \phi_2 = \alpha/(1+\alpha) \), \( \phi_3 = \psi/(1+\alpha) \), \( \xi(d_t) = \psi d_t \) with \( \psi \geq 0 \), \( \Theta = \alpha/(1+\alpha) \) and \( \nu_t \) is the error term.

We solve the model under the assumption of weak rationally (Chow, 1983), according to which the conditional forecast error of the exchange rate is conjectured to depend on unexpected events that occur between \( t \) and \( t+j \), that is

\[
e_{t+j} - e_{t+j}^* = u_{t+j} + \Gamma_1 u_{t+j-1} + \Gamma_2 u_{t+j-2} + \cdots + \Gamma_j u_{t} + \Phi_0 (\zeta_{t+j} - \zeta_{t+j}^*) + \Phi_1 (\zeta_{t+j-1} - \zeta_{t+j-1}^*) + \Phi_2 (\zeta_{t+j-2} - \zeta_{t+j-2}^*) + \cdots + \Phi_{j-1} (\zeta_{t+1} - \zeta_{t+1}^*) + \Phi_j (\zeta_t - \zeta_t^*)
\]  

(6)

For \( j=1 \), the above expression becomes:

\[
e_{t+1} - e_{t+1}^* = u_{t+1} + \Gamma_1 u_{t} + \Phi_0 (\zeta_{t+1} - \zeta_{t+1}^*)
\]  

(7)

Subtracting the term \( \Theta e_{t+1} \) from both sides of equation (5) and using (7), yields:
\[ e_t - \Theta e_{t-1} = \phi' \zeta_t - \Phi (e_{t+1} - e_{t+1}) + u_t = \phi' \zeta_t - \Phi (\zeta_{t+1} - \zeta_{t+1}) - \Theta u_{t+1} - \Phi \Gamma u_t + u_t \]

or,
\[ e_{t+1} = \frac{1}{\Theta} e_t - \frac{\phi'}{\Theta} \zeta_t + \Phi (\zeta_{t+1} - \zeta_{t+1}) + u_{t+1} - \left( \frac{1}{\Theta} - \Gamma \right) u_t \]

or,
\[ e_t = \frac{1}{\Theta} e_{t-1} - \frac{\phi'}{\Theta} \zeta_{t-1} + \Phi (\zeta_t - \zeta_t) + u_t - \sigma u_{t-1} \quad (8) \]

where \( \sigma = (1/\Theta) - \Gamma. \)

Subtracting the term \( e_{t-1} \) from both sides and assuming that the fundamentals follow a driftless random walk process, equation (8) becomes a conventional EC mechanism with a MA(1) error process:
\[ \Delta e_t = \lambda_1 e_{t-1} + \lambda_2 \Delta \zeta_{t-1} + \Phi \Delta \zeta_t + \eta_t \quad (9) \]

where \( \lambda_1 = (1/\Theta) - 1, \lambda_2 = -\phi'/\Theta \) and \( \eta_t = u_t - \sigma u_{t-1}. \) If the growth rates of \( e_t \) and \( \zeta_t \) are zero, then the long-run solution of equation (9) becomes:
\[ e_t = \phi'/(1 - \Theta) \zeta_t. \]

3. Empirical Analysis

We use quarterly US and euro area observations from 1999Q1 to 2007Q1 on the average exchange rate, defined as dollars per euro, narrow money supplies, real incomes and the US federal government debt held by foreign and international investors. All the data were obtained from the International Statistical Yearbook CD-ROM with the exception of the debt which was obtained from St. Louis FED data base. The marked decrease of the exchange rate up to the end of 2001 and a substantial increase thereafter is obvious from Figure 1. The unit root test proposed by Zivot and Andrews (1992) was
employed. The latter is more robust than the traditional Dickey and Fuller (DF) test in the presence of a structural break. The estimated break date occurs at the fourth quarter of 2001, but the unit root hypothesis is not rejected, implying that the exchange rate cannot be characterized as a trend-stationary process with one time shift in the slope of the trend function. The slope dummy is highly significant, indicating that the exchange rate has experienced a change in the slope. In addition, the DF unit root test has indicated that all the other variables concerned can be characterized as integrated processes.

The fully modified OLS estimates of the Phillips and Hansen (1990) co-integration analysis reported in Table 1 indicate that the debt-augmented monetary model is validated as a long-run equilibrium relationship, in the sense that a co-integrating relationship exists among the exchange rate, the relative money supply, the relative real income and the debt/GDP ratio. The estimated long-run parameters have signs consistent with the theory and the proportionality postulate between the exchange rate and the monetary fundamentals is not rejected by the data. The co-integrating residuals from the restricted exchange rate model, after imposing the homogeneity restrictions, which are plotted in Figure 2 are stationary and serially uncorrelated as indicated by the DF unit root test and the Box-Pierce $Q$ test for serial correlation.
Having established a co-integrated relationship between the exchange rate and macroeconomic fundamentals, we then proceed to examine the associated EC mechanism which describes the short-run dynamics. The maximum likelihood (ML) estimation results are reported in Table 2. The growth rate of the relative money supply significantly affects the growth rate of the exchange rate with a sign consistent with theory. The homogeneity restriction is supported by the data, implying that the monetarist postulate is valid not only in the long-run but also in the short-run. One implication of this finding is that the monetary conditions are important in explaining the behaviour of the euro/dollar exchange rate. The growth rate of the relative real income enters positively and significant, implying that if the growth rate of the Euro area exceeds the corresponding growth rate of the USA the euro will appreciate against the dollar. The growth rate of the debt enters positively with a highly significant coefficient, indicating that an increase in the US international debt will appreciate the euro against the dollar. This result is
consistent with debt-augmented monetary model, presented in Section 2. The EC term has the expected sign and its size indicates that any deviations from the systematic relationship are corrected immediately corrected. The estimated residuals are stationary and serially uncorrelated as indicated by the DF unit root test and the Box-Pierce $Q$ test. Figure 3 plots the actual and fitted values of the EC model. The value of $R^2$ suggests that 58% of the short-run variation in the dollar exchange rate of the euro is explained by the model.

Overall, the relative real income is the most economically significant determinant, whereas the US international debt is the most statistically significant determinant of the euro/dollar exchange rate.

![Figure 3: Actual and fitted values of the error correction exchange rate model](image-url)
REFERENCES


Table 1. Fully modified OLS Estimates of the Exchange Rate Equations

\[ e_t = \theta_1 (m_t^* - m_t) + \theta_2 (y_t - y_t^*) + \theta_3 d_t + \theta_4 dum + \varepsilon_t, \]

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<tr>
<td>1,17</td>
<td>1,98</td>
<td>0,49</td>
<td>0,03</td>
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<tr>
<td>(3,12)***</td>
<td>(1,91)***</td>
<td>(2,57)***</td>
<td>(2,13)***</td>
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Wald test:
\[ \theta_j = 1 : \chi^2(1) = 0,20[0,66]; \ \theta_2 = 1 : \chi^2(1) = 0,90[0,34]; \ \theta_3 = \theta_4 : \chi^2(1) = 0,50[0,48] \]

AIC: ADF(1) = -3,83***; SBC: DF = -3,37***; Q(30) = 19,89[0,92]

\[ e_t = \zeta_1 [(m_t^* - m_t) + (y_t - y_t^*)] + \zeta_2 d_t + \zeta_3 dum + \varepsilon_t, \]

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<tr>
<td>1,30</td>
<td>0,39</td>
<td>0,03</td>
<td></td>
</tr>
<tr>
<td>(3,84)***</td>
<td>(3,02)***</td>
<td>(3,00)***</td>
<td></td>
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Wald test:
\[ \zeta_1 = 1 : \chi^2(1) = 0,78[0,38] \]

AIC: ADF(1) = -3,83***; SBC: DF = -3,46***; Q(30) = 19,87[0,82]

\[ e_t - (m_t^* - m_t) - (y_t - y_t^*) = \nu_1 d_t + \nu_2 dum + \varepsilon_t, \]

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<tr>
<td>0,51</td>
<td>0,02</td>
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<tr>
<td>(6,27)***</td>
<td>(6,32)***</td>
</tr>
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AIC: ADF(1) = -3,81***; SBC: DF = -3,37***; Q(30) = 18,80[0,94]

All equations have included an intercept, which is statistically significant in all cases. The variable \( dum \) is a slope dummy, which takes on the value one from 2002Q1 to 2007Q1 and zeros elsewhere. The numbers in parentheses are \( t \)-ratios and the numbers in square brackets are \( p \)-values. The Phillips-Hansen estimates have used Bartlett weights with a truncation lag=1. The same results obtained with truncation lag=4. The Dickey-Fuller regressions have included an intercept. AIC and SBC are the Akaike Information Criterion and Schwartz Bayesian Criterion. Box-Pierce Q test for serial correlation. ***, ** indicate significance at 1% and 5%, respectively.
Table 2. Maximum Likelihood Estimates of the error correction exchange rate equation: $\Delta e_t = -\gamma e_{t-1} + \phi \Delta z_t + \eta_t$

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>St. Error</th>
<th>T-ratio[p-value]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{t-1}$</td>
<td>-0.9708</td>
<td>0.2589</td>
<td>-3.75[0.00]***</td>
</tr>
<tr>
<td>$\Delta (m_t - m_r)$</td>
<td>0.8195</td>
<td>0.3629</td>
<td>2.26[0.03]**</td>
</tr>
<tr>
<td>$\Delta (y_t - y_r)$</td>
<td>3.3092</td>
<td>0.7932</td>
<td>4.17[0.00]***</td>
</tr>
<tr>
<td>$\Delta d_t$</td>
<td>0.5140</td>
<td>0.0953</td>
<td>5.40[0.00]***</td>
</tr>
</tbody>
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$R^2$-Bar=0.5826; SEE=2.57; DW=1.63;

Parameters of the MA error specification: $\eta_t = \mu_t + 0.8602 \mu_{t-1}$

$\begin{pmatrix} 3.3898 \end{pmatrix}$***

Univariate tests for the error term:

**DF unit root test**=$-4.34$

**Box-Pierce Q(30) test**=12.1909[0.998]

**Wald tests:**
EC term $\gamma$=$-1$: $\chi^2(1)$=0.0128[0.91]
Homogeneity restriction on relative money supply: $\chi^2(1)$=0.2475[0.62]

The DF unit root test includes a constant and a time trend. The truncation lag was selected by reference to AIC and SBC. Numbers in square brackets are p-values. The 95% critical value of the DF test is equal to -3.59. *** indicates significance at 1% and ** indicates significance at 5%.